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Human Capital, Wage Inequality, and Economic Growth

Advisor: Prof. Hsun Chu

Student: Shue-Nam Yiu

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Human Capital, Wage Inequality, and Economic Growth

業經本委員會審議通過

論文口試委員會委員：張詠華

陳平河

朱廷

論 文 指 導 教 授 ：朱廷

經 濟 系 系 主 任 ：劉志同

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姚澍楠　謹誌

東海大學經濟研究所

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Abstract

This paper develops an endogenous growth model in which human capital and public infrastructure are the engines of economic growth, and in which there are two types of individuals: the “poor” provides low-skilled labor, while the “rich” provides high-skilled labor. The government collects income tax revenues to run a training program to promote the accumulation of human capital that improves the productivity of high-skilled labor and provides public infrastructure to enhance the final-good production. It also allocate a part of tax revenues on wasteful spending. We examine the effects of the policies (training policy and tax policy) on wage inequality, growth, and welfare. Our main results are as follows. First, suppose the government allocates its tax revenues between training program and infrastructure expenditure, increasing the proportion of infrastructure expenditure has a positive effect on growth and welfare. Second, raising the income tax rate can stimulate growth and improve welfare. Finally, as these policies boost growth, they also worsen wage inequality, implying that there is a trade-off between economic development and wage equality.

Keyword: human capital, endogenous growth, wage inequality, welfare

摘要

本文建構一個以人力資本累積與公共基礎建設作為成長引擎的內生成長模型。經濟體系中存在兩種人：窮人提供低技術勞動力而富人提供高技術勞動力。政府對富人課徵所得稅，將稅收用來培訓高所得勞工以幫助其累積人力資本，同時稅收也用來提供可增加最終財廠商生產力的基礎建設，此外，政府會把一部分稅收投放在浪費性支出上。本文將探討政府政策(如所得稅率、基礎建設的支出比例)對經濟成長及社會福利的影響。我們有以下發現。第一，當政府必須將稅收分配在高技術勞工培訓與公共基礎建設，增加公共基礎建設的比例會降低經濟成長與社會福利；第二，提高所得稅稅率對經濟成長與社會福利有正面影響；第三，當政府的政策提高經濟成長率，同時也會使工資不均問題惡化，即經濟發展與所得分配具有抵換關係。

關鍵字：人力資本、內生成長、工資不均、社會福利

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1. Introduction

Human capital is one of the key determinants of economic growth. A substantial body of literature has argued that human capital accumulation leads to an increase in economic growth. Among the important contributions, Lucas (1988) constructs an endogenous growth model emphasizing specialized human capital accumulation through schooling and learning-by-doing. Lucas uses the model to investigate the optimal subsidy on human capital accumulation that corrects the externality and induces households to invest the socially optimal amount in human capital. Mankiw (1992) augments the Solow (1956) model by including accumulation of human capital as well as physical capital, and Mankiw (1995) finds that various measures of human capital, such as enrollment rate in primary and secondary schools, are positively associated with economic growth.

Another feature we consider is the effect of infrastructure on productivity. Aschauer (1989) studies the relationship between aggregate productivity and stock and flow government-spending variables. Their result shows that a core infrastructure of streets, highways, airports, mass transit, sewers, water systems, have most explanatory power for productivity. Regardless of whether it is theoretically or intuitively, infrastructure has an effect of raising the productivity of labor. In Taiwan, infrastructure development program is the primary policy. For example, the “Forward-looking Infrastructure Development Program” focuses on improving infrastructure such as transportation, water environments, and green energy, etc.

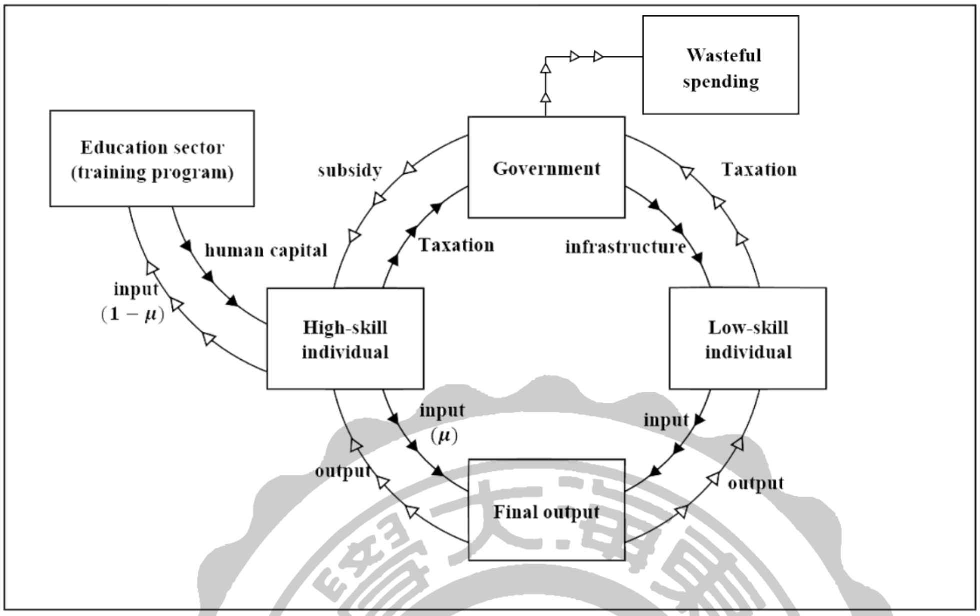

Figure 1: the concept of this model

This paper develops an endogenous growth model in which human capital and public infrastructure are the engines of economic growth, and in which there are two types of individuals: the "poor" provides low-skilled labor, while the "rich" provides high-skilled labor. The government runs a training program to promote the accumulation of human capital that improves the productivity of high-skilled labor. Within this framework, we examine the effects of the policies (training policy and tax policy) on wage inequality, growth, and welfare. We assume that higher wage inequality decreases households' welfare. This effect has been empirically examined by, for example, Krueger and Perri (2003), who investigate the welfare consequences of the stark increase in wage and earnings inequality, and find that about 60 percent of US households face welfare losses.

Figure 1 shows the structure of our model. The household sector has two different type of households, high-skilled and low-skilled households. The firm hires both high-skilled and low-skilled households as labor inputs to produce the final output. The government collects income taxes from both high-skilled and low-skilled labor, and then allocates the tax revenues between the training of students (to promote the accumulation of human capital), providing infrastructure expenditure and wasteful spending.

Our study is closely related to recent studies on education and endogenous growth. Chakraborty and Gupta (2009) assume that both rich and poor individuals can accumulate their own knowledge, but the knowledge needs to trickle down from more knowledgeable (rich) persons to inferiors (poor). The government imposes a proportional income tax on rich individuals and uses the tax revenue to finance the educational subsidy given to poor individuals. Their study focuses on the examination of the optimal educational subsidy policy. In addition, Mattalia (2012) constructs an R&D-based endogenous growth model, and shows that the productivity of schooling affects the long run growth of the economy, contrary to the productivities of the other sectors (i.e., final good sector). Dias and Tebaldi (2012) construct an education sector to provide human capital accumulation. Their model shows that improvements in the quality of institutions foster human capital accumulation and decrease income inequality. In particular, our analysis is mostly related to Greiner (2008), who considers two types of households, one of which acquires human capital or skills through education, while the other remains unskilled. Greiner investigates the effect of fiscal policy on growth and welfare. Their primary focus, however, is on the relationship between human capital and growth, while our analysis concerns the utility/disutility of wage inequality.

The dissertation is structured as follows. Section 2 describes the model structure. Section 3 performs a numerical simulation to analyze how the government policy affects economic growth and social welfare. Finally, some conclusion is drawn in Section 4.

2. The model

We consider three sectors in our economy: a household sector, a productive sector, and the government sector. In the household sector, there are two different type of households – high-skilled household and low-skilled household. The perfectly competitive final-good firms produce a single final good using both types of labor. The government collects income taxes from high-skilled labor, and then allocates the tax revenues between the training of students and providing infrastructure expenditure. In what follows, we describe them in turn.

2.1 Households

The economy is populated by two representative infinitely-lived households. The first one is the high-skilled (rich) household, which is endowed with H unit of high-skilled labor that is inelastically supplied. High-skilled labor is allocated between the final good sector, denoted by H_Y, and the educational sector, denoted by H_E. Thus,

$$H = H_Y + H_E. \tag{1}$$

The second one is the low-skilled (poor) household, which is endowed with L unit of low-skilled labor that is inelastically supplied only in the final food sector. We drop the time index for simplicity and use the subscript i where $i = H$ denotes the variables corresponding to the rich household and $i = L$ denotes the variables corresponding to the poor household. Both types of households can accumulate physical capital, K_i and maximize their discounted stream of utility resulting from consumption, C_i over an infinite time horizon subject to the budget constraint.

The representative high-skilled household's discount lifetime utility is given by:

$$W_H = \int_0^{\infty} (lnC_H + \varphi_H w_p)\, e^{-\rho t} dt, \tag{2}$$

Where the parameter $\rho > 0$ is the constant rate of time preference. We introduce the parameter $\varphi_i \in (-\infty, \infty), i = H, L$, which determines the disutility/utility of wage inequality. w_p represents the level of inequality (to be detailed later). The utility is increasing in consumption C_i and decreasing or increasing in wage inequality w_p.

The representative high-skilled household's discount lifetime utility subject to:

$$\dot{K}_H + C_H = rK_H + (1 - \tau_H)\omega_H H, \tag{3}$$

The overdot denotes the rate of change with respect to time. $\tau_H \in (0,1)$ is the income tax rate levied on the high-skilled household. ω_H is the wage rate of high-skilled labor and r is the rate of return on physical capital.

The representative high-skilled household maximizes Eq. (2) subject to Eq. (3) by choosing consumption, C_H and physical capital, K_H. The current value Hamiltonian of high-skilled household is given by:

$$\mathcal{H}^H = ln[C_H] + \varphi_H w_p + \lambda(rK_H + (1 - \tau_H)\omega_H H)$$

Necessary optimality conditions are given by:

$$\frac{1}{C_H} = \lambda_H, \tag{4a}$$

$$r\lambda_H = -\dot{\lambda}_H + \rho\lambda_H \tag{4b}$$

where λ_H is the co-state variable of K_H. We assume that the transversality condition

$\lim_{t\to\infty} e^{-\rho t}\lambda_H K_H = 0$ holds.

Using Eqs. (4a) and (4b) we can obtain the growth rate of consumption:

$$\frac{\dot{C}_H}{C_H} = r - \rho. \tag{5}$$

Likewise, the representative low-skilled household's life-time utility function is specified as:

$$W_L = \int_0^\infty (lnC_L + \varphi_L w_p)\, e^{-\rho t} dt, \tag{6}$$

subject to:

$$\dot{K}_L + C_L = rK_L + (1 - \tau_L)\omega_L L, \tag{7}$$

where ω_L is the wage rate of low-skilled labor. $\tau_L \in (0,1)$ is the income tax rate levied on the low-skilled household.

Follow the previous step, the optimality conditions for the problem of the low-skilled household are given by:

$$\frac{1}{C_L} = \lambda_L, \tag{8a}$$

$$r\lambda_L = -\dot{\lambda}_L + \rho\lambda_L, \tag{8b}$$

and the usual Keynes-Ramsey rule:

$$\frac{\dot{C}_L}{C_L} = r - \rho. \tag{9}$$

Eqs. (5) and (9) show that consumptions of two types of households grow at a common rate. The aggregate consumption is defined as:

$$C = C_H + C_L. \tag{10}$$

2.2 Final good sector

The final good sector is perfect competitive. There is one representative firm in the productive sector that hires physical capital and two types of labor to produce a single final good Y. The production function of the representative firm is given by:

$$Y = AK^{1-\alpha}((h_e H_Y)^{\beta} L^{1-\beta})^{\alpha} G^{v},\ 0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < v < 1 \tag{11}$$

H_Y and L are the high-skilled and low-skilled labor, respectively, used in the final good sector. $A > 0$ is a productivity parameter. h_e is the level of human capital associated with the high-skilled labor. G is the government's infrastructure expenditure that has a positive effect on the productivity of final good production. To ensure sustained growth, we assume that $v = (1 - \beta)\alpha$.

Under the assumption of perfect competition, the factor price will equal to its marginal productivity. Standard profit maximization gives the following first-order conditions:

$$r = (1 - \alpha) Y K^{-1}, \tag{12a}$$

$$\omega_H = \alpha\beta Y H_Y^{-1}, \tag{12b}$$

$$\omega_L = \alpha(1-\beta) Y L^{-1}. \tag{12c}$$

Now, we can define wage inequality from Eqs. (12b) and (12c). The level of wage inequality is defined as the ratio of the wages paid to high-skilled workers relative to wages paid to low-skilled workers, i.e., the wage premium, which is given by:

$$w_p = \frac{\omega_H}{\omega_L} \tag{12d}$$

2.3 Human capital accumulation

2.3 Human capital accumulation

We follow Greiner (2008) to assume that the law of motion of per-capita human capital at the economy-wide level is given by:

$$\dot{h}_e = \epsilon {H_E}^{\psi} h_e. \tag{13}$$

As shown in Eq. (13), human capital accumulation $\dot{h}_e$ is positively related to the high-skilled labor that the government hires to conduct training and education, H_E, and positively related to the current level of human capital h_e. The parameter $\epsilon \in (0,1)$ is a productivity parameter and the parameter $\psi \in (0,1)$ denotes the efficiency of the training labor.

2.4 The government

The government collects income taxes from high-skilled labor income and then uses the tax revenues (i) to hire high-skilled labor to develop human capital, (ii) to finance the infrastructure expenditure, G, and (iii) to finance the wasteful government spending, M. Examples of wasteful government expenditure include government payment for the principal and interest of debts and general administration expenditure[1] Thus, the government's flow budget constraint can be expressed as:

$$T = \omega_H H_E + G + M, \tag{14}$$

where $T = \tau_H \omega_H H + \tau_L \omega_L L$ denotes the tax revenues.

We assume that the infrastructure expenditure and wasteful government spending are exogenous fractions of the tax revenues, i.e., $G = \theta_G T$, $M = \theta_M T$.[2] Moreover, we denote μ as the fraction of high-skilled labor employed in the final good sector, and $1-\mu$ the fraction of high-skilled labor employed in the education sector, i.e., $H_Y = \mu H$ and $H_E = (1-\mu)H$.[3] After putting these expressions into Eq. (14), we can obtain:

$$\mu = \frac{\beta[1-\tau_H(1-\theta_G-\theta_M)]}{\beta[1-\tau_L(1-\theta_G-\theta_M)]+\tau_L(1-\theta_G-\theta_M)}. \tag{15}$$

Combining Eqs. (1), (3), (7) and (14), the social resource constraint can be written as

$$\dot{K} = Y - C - G - M. \tag{16}$$

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1 T. Ihori and C.C. Yang (2012) point out that the government not only provides useful public goods but also engages in wasteful spending, i.e., political spending.

2 Where $(\theta_G + \theta_M) < 1$.

3 Note that μ is endogenously determined in our model.

2.5 The balanced growth path and equilibrium

The equilibrium of the economy is defined as a set of the factor prices (r, ω_H, ω_L), a set of aggregate allocations (K_H, K_L, C_H, C_L, G), such that the high-skilled and low-skilled households maximize their life-time utility, the final-good firm maximizes its profit, and the government budget constraint (14) holds.

Combining Eqs. (10) and (12a), the growth rate of aggregate consumption can be expressed as:

$$\frac{\dot{C}}{C} = (1-\alpha)AK^{-\alpha}\left((h_e H_Y)^{\beta}(GL)^{1-\beta}\right)^{\alpha} - \rho. \tag{17}$$

By using the resource constraint [Eq. (15)] and the evolve function of human capital [Eq. (13)], the growth rate of aggregate physical capital and human capital are respectively given by

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} - \frac{G}{K} - \frac{M}{K}, \tag{18}$$

$$\frac{\dot{h}_e}{h_e} = \epsilon((1-\mu)H)^{\psi}. \tag{19}$$

Along the balanced growth path (BGP), all growing variables grow at a common rate, which we define as $\tilde{\gamma}$. Thus we have:

$$\tilde{\gamma} = \tilde{\gamma}_{h_e} = \tilde{\gamma}_K = \tilde{\gamma}_C,$$

where $\tilde{\gamma}_X$ is the growth rate of generic variables $X = h_e, K, C$ along the BGP. It is also useful to define the following transformed variables:

$z \equiv \frac{C}{K},\ h \equiv \frac{h_e}{K},\ g \equiv \frac{G}{K},\ w_H \equiv \frac{\omega_H}{K},\ w_L \equiv \frac{\omega_L}{K}.$

Combining the transformed variables into Eq. (17), the growth rate of aggregate consumption can be expressed as:

$$\frac{\dot{C}}{C} = A(1-\alpha)\left(gL\left(\frac{h\mu H}{gL}\right)^{\beta}\right)^{\alpha} - \rho. \tag{20}$$

After some algebra, the macroeconomy can be expressed by the set of equations:

$$\tilde{\gamma} = (1-\alpha)A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha} - \rho, \tag{21a}$$

$$\tilde{\gamma} = A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha} - \tilde{z} - \tilde{g} - \theta_M\frac{T}{K}, \tag{21b}$$

$$\tilde{\gamma} = \epsilon((1-\tilde{\mu})H)^{\psi}, \tag{21c}$$

$$\tilde{w}_H = \alpha\beta\mu^{-1}H^{-1}\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{21d}$$

$$\tilde{w}_L = \alpha(1-\beta)L^{-1}A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{21e}$$

$$\tilde{r} = (1-\alpha)A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{21f}$$

$$\tilde{\mu} = \frac{\beta[1-\tau_H(1-\theta_G-\theta_M)]}{\beta[1-\tau_L(1-\theta_G-\theta_M)]+\tau_L(1-\theta_G-\theta_M)}, \tag{21g}$$

$$\alpha A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha}(\tau_H\beta\tilde{\mu}^{-1} + \tau_L(1-\beta)) = \tau_H\alpha\beta A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha}\tilde{\mu}^{-1}(1-\tilde{\mu}) + \tilde{g} + \theta_M\frac{T}{K}. \tag{21h}$$

in which eight unknowns, $\tilde{\gamma},\ \tilde{\mu},\ \tilde{r},\ \tilde{h},\ \tilde{z},\ \tilde{g},\ \tilde{w}_H,\ \tilde{w}_L$, are determined. In Eqs. (21) we use the notation “~” to denote variables at their steady-state values. Due to its complexity, we solve the model through the numerical simulation in the next section.

3. Numerical Results

In this section we perform numerical simulations to provide a quantitative illustration on the growth and social welfare effects of policy. We first assign parameter values of the model. First, we set the constant rate of time preference to a standard value $\rho = 0.05$ (Acemoglu and Akcigit, 2012), and the capital share is set to 0.4, i.e., $\alpha = 0.6$ (Elsby et al., 2013). The share of high-skilled labor is set to $\beta = 0.6$ and the productivity parameter of human capital formation is set to $\epsilon = 0.05$, which gives us a skill premium around 1.6 (Angelopoulos et al., 2015). The productivity parameter in final-good production A, aggregate labor supply of high-skilled labor H, and aggregate labor supply of low-skilled labor L are all normalized to unity without loss of generality. The efficiency of the training labor in accumulating human capital is set to $\psi = 0.5$. This parameter is mainly calibrated such that the balanced growth rate is around 5%.

For the income tax rate, according to the National Tax Administration of Taiwan, the tax rate of high-income individual is about 30% and low-income individual is around 10%; thus we set the value $\tau_H = 0.3$ and $\tau_L = 0.1$. For the magnitude of the infrastructure expenditure, we use the baseline value $\theta_G = 0.2$, but we will observe different level of θ_G, given that a major purpose of this analysis is to explore the effect of different degrees of government infrastructure expenditure. Furthermore, as we defined the wasteful government spending above, we assume

$$\theta_M = \frac{\text{government payment for the principal \& interest of debts} + \text{general administration expenditure}}{\text{total government expenditure}}$$

According to the Directorate General of Budget, Accounting and Statistics,

Table 1
Baseline parameters

Parameter	Value	Parameter	Value
ρ	0.05	ϵ	0.05
α	0.6	ψ	0.5
β	0.6	A	1
L	1	τ_H	0.3
H	1	τ_L	0.1
φ_H	0.05	θ_G	0.2
φ_L	-0.06	θ_M	0.1

Executive Yuan, the magnitude of the wasteful government spending between 2011 and 2016 are around 10%; thus we set $\theta_M = 0.1$.

Furthermore, Krueger and Perri (2003) measure the welfare effect on wage inequality under different earnings group. Their results show that the lowest earning group losses around 6% welfare implied by the increase in earnings inequality. By contrast, the highest earning group gain 5% welfare. Accordingly, we set $\varphi_L = -0.06$ and $\varphi_H = 0.05$. Table 1 summarizes the parameter values in our numerical analysis.

3.1 Growth effect

Table 2 reports the effects for the model based on the benchmark parameters value given in Table 1. Table 2 clearly shows that, a larger fraction of infrastructure expenditure depresses economic growth. The intuition can be explained as follows. The government allocates the tax revenues among developing human capital, providing infrastructure expenditure, and wasteful government spending. As the proportion of infrastructure expenditure increases, the resource to develop human capital will fall. While both human capital and infrastructure expenditure contribute positively to

Table 2

The balance growth rate on different government expenditure

θ_G	$\tilde{\gamma}$
0.2	**0.02476***
0.1	0.02639
0.3	0.02300

*: the benchmark result

economic growth, it appears in our model that the effect of decreasing human capital dominates. In our model, however, both infrastructure and training program policy are the engine of our economic growth, it reveals that the growth effect of infrastructure expenditure is ambiguous [$\left(\mu\frac{\partial\theta_g}{\partial g}\right)^{\beta}+\left(g\frac{\partial\theta_g}{\partial\mu}\right)^{1-\beta}\gtrless 0$]. As a consequence, when the fraction of infrastructure expenditure $\theta_G = 0.2$ raises to 0.3, it leads to a decrease in the balance growth rate. On the other hand, when $\theta_G = 0.2$ change to 0.1, it will boosts economic growth which means in our benchmark scenario, the human capital effect is higher than public infrastructure effect ($\frac{\partial\gamma}{\partial\theta g} < 0$).

In terms of how the tax policy affects economic growth, Table 3 reports results of raising the taxes τ_H and τ_L under the benchmark scenario. It shows that the balance growth rate rises as the tax rate of both high-income and low-income individuals rise regardless of whether the proportion of government infrastructure expenditure is high or low. Intuitively, when the tax rates increase[4], tax revenues directly increases.

[4] Note that high-skilled and low-skilled labors are inelastically supplied in our model. Therefore, the labor tax has an effect that is quite similar to the lump-sum tax, i.e., it has no additional distortion.

Table 3

Changes in τ_H, τ_L, θ_G, and growth effect

τ_H	τ_L	$\tilde{\gamma}$	w_p	τ_H	τ_L	$\tilde{\gamma}$	w_p
		$\theta_G = 0.2$				$\theta_G = 0.2$	
0.3	0.1	**0.02476***	**1.98734***	0.3	0.1	**0.02476***	**1.98734***
0.2	0.1	0.0207	1.8108	0.3	0.05	0.0239	1.9430
0.4	0.1	0.0279	2.1806	0.3	0.2	0.0263	2.0760
0.5	0.1	0.0308	2.4154	0.3	0.3	0.0279	2.1766
		$\theta_G = 0.1$				$\theta_G = 0.1$	
0.3	0.1	0.0264	2.0790	0.3	0.1	0.0264	2.0790
0.2	0.1	0.0297	2.3210	0.3	0.05	0.0255	2.0263
0.4	0.1	0.0328	2.6333	0.3	0.2	0.0280	2.1842
0.5	0.1	0.0225	1.8810	0.3	0.3	0.0293	2.2884

*: the benchmark result

The government therefore has more resources to develop human capital and public infrastructure. Since human capital growth effect higher than public infrastructure, the balanced growth rate is stimulated in response. This can also be seen mathematically by referring to Eqs. (21c) and (21g). From (21g) we can obtain $\frac{\partial \mu}{\partial \tau_i} < 0, i = H, L$. This together with (21c) implies that $\frac{\partial \tilde{\gamma}}{\partial \tau_i} > 0, i = H, L$.

So far, we know a rise of both taxes boosts economic growth. Now we turn to the effects on wage inequality. It can be seen in Table 3 that higher taxes raise the wage gap between high-skilled and low-skilled workers. The intuition is easy to understand, when the labor taxes increase, higher tax revenues mean that the government devoted more resources in hiring high-skilled workers to develop human capital. Accordingly, the demand for high-skilled workers increases. This will raise the wage of high-skilled workers, and in turn worsen the wage inequality.

In addition, Table 3 shows that when $\theta_G = 0.1$ raises to 0.2, w_p decreases, which indicates that wage inequality is mitigated when the fraction of infrastructure expenditure is higher. The intuition can be explained as follows. When the fraction of infrastructure expenditure is higher, the portion of the tax revenues to hire high-skilled workers is smaller. Therefore, the demand for high-skilled workers will decrease as a response. In contrast to the effect mentioned above, the wage of high-skilled workers falls, and thus the wage inequality mitigates.

3.2 Welfare

Besides the economic growth, we also investigate another key issue which concerns the effects of government policy on welfare. The aggregate welfare is assumed to be utilitarian:

$$W = W_H + W_L \tag{22}$$

Where W represent the aggregate welfare.

Eqs. (2) and (6) represent the welfare of high-skilled and low-skilled individual, respectively, and depend on the consumption, $C_i, i = H, L$. Further, in our model both consumptions of high-skilled and low-skilled growth at a common rate γ. Therefore, the consumption of high-skilled and low-skilled individual can be expressed as:

$$C_t^H = C_0^H e^{\gamma_c t} \tag{23}$$

$$C_t^L = C_0^L e^{\gamma_c t} \tag{24}$$

where C_0^H and C_0^L represent the initial consumption of high-skilled and low-skilled individual, respectively.

To obtain the initial consumption, we define the transformed variables with $z_H \equiv$

$\frac{C_H}{K_H}$ and $z_L \equiv \frac{C_L}{K_L}$. Using definition of $z_H \equiv \frac{C_H}{K_H}$ and Eqs. (3), (11) and (20) we can obtain the following equations:

$$\frac{\dot{K}_H}{K_H} = r + (1-\tau_H)\frac{\omega_H}{K_H}H - \frac{C_H}{K_H} \tag{25a}$$

$$\frac{C_H}{K_H} = r + (1-\tau_H)\frac{\omega_H}{K_H}H - \gamma \tag{25b}$$

$$C_0^H = (r + (1-\tau_H)\frac{\omega_H}{K_0^H}H - \gamma_c)K_0^H \tag{25c}$$

$$C_0^H = (r + (1-\tau_H)\alpha\beta\frac{K_0}{K_0^H}A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha}\mu^{-1} - \gamma_c)K_0^H \tag{25}$$

with K_0^H is the initial level of physical capital stock of high-skilled individual.

Inserting Eq. (23) into Eq. (2), the welfare function of high-skilled individual is given by[5]:

$$W_H = \int_0^{\infty}[lnC_0^H + \gamma_c t + \varphi_H w_p]\, e^{-\rho t}dt$$

$$= \frac{1}{\rho}(lnC_0^H + \varphi_H w_p) + \frac{1}{\rho^2}\gamma_c \tag{26}$$

Similarly, we can obtain the initial consumption equation by using the definition of $z_L \equiv \frac{C_L}{K_L}$, Eqs. (7), (11) and (20) and welfare function of low-skilled individual by inserting Eq. (24) into Eq. (6). The initial consumption equation and welfare function of low-skilled individual are respectively given by :

[5] See Appendix A

$$C_0^L = (r + \alpha(1-\beta)(1-\tau_L)\frac{K_0}{K_0^L}A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha} - \gamma_c)K_0^L. \tag{27}$$

$$W_L = \int_0^{\infty}[lnC_0^L + \gamma_c t + \varphi_L w_p]\, e^{-\rho t}dt$$

$$= \frac{1}{\rho}(lnC_0^L + \varphi_L w_p) + \frac{1}{\rho^2}\gamma_c \tag{28}$$

where K_0^L is the initial level of physical capital stock of low-skilled individual.

Now, we are ready to analyze the welfare effect by inserting Eqs. (22), (25), (26), (27), and (28) into the expression for the numerical simulations. The expressions are given by:

$$\tilde{\gamma} = (1-\alpha)A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha} - \rho, \tag{a}$$

$$\tilde{\gamma} = A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha} - \tilde{z} - \tilde{g} - \theta_M\frac{T}{K}, \tag{b}$$

$$\tilde{\gamma} = \epsilon((1-\tilde{\mu})H)^{\psi}, \tag{c}$$

$$\tilde{w}_H = \alpha\beta\mu^{-1}H^{-1}\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{d}$$

$$\tilde{w}_L = \alpha(1-\beta)L^{-1}A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{e}$$

$$\tilde{r} = (1-\alpha)A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}, \tag{f}$$

$$\tilde{\mu} = \frac{\beta[1-\tau_H(1-\theta_G-\theta_M)]}{\beta[1-\tau_L(1-\theta_G-\theta_M)]+\tau_L(1-\theta_G-\theta_M)}, \tag{g}$$

$$\alpha A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha}(\tau_H\beta\tilde{\mu}^{-1}+\tau_L(1-\beta))=\tau_H\alpha\beta A(\tilde{g}L(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L})^{\beta})^{\alpha}\tilde{\mu}^{-1}(1-\tilde{\mu})+\tilde{g}+\theta_M\frac{T}{K}. \quad \text{(h)}$$

$$W = W_H + W_L \quad \text{(i)}$$

$$\tilde{W}_H = \frac{1}{\rho}ln(C_0^H) + \frac{1}{\rho^2}\tilde{\gamma} \quad \text{(j)}$$

$$\tilde{W}_L = \frac{1}{\rho}ln(C_0^L) + \frac{1}{\rho^2}\tilde{\gamma} \quad \text{(k)}$$

$$C_0^H = (r + (1-\tau_H)\alpha\beta\frac{K_0}{K_0^H}A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha}\mu^{-1} - \tilde{\gamma})K_0^H \quad \text{(l)}$$

$$C_0^L = (r + (1-\tau_L)\alpha(1-\beta)\frac{K_0}{K_0^L}A\left(\tilde{g}L\left(\frac{\tilde{h}\tilde{\mu}H}{\tilde{g}L}\right)^{\beta}\right)^{\alpha} - \tilde{\gamma})K_0^L \quad \text{(m)}$$

In this numerical simulations, there have three additional exogenous parameter – the initial aggregate physical capital stock K_0, the initial physical capital stock of high-skilled individual K_0^H and low-skilled individual K_0^L. To simplify the analysis, we set $K_0 = 1$. Further, we have assumed $K_0^H > K_0^L$ that the origination of the physical capital stock owned by the high-skilled households is larger than the low-skilled households as mentioned above, so we set K_0^H and K_0^L as 0.65 and 0.35, respectively.

Table 4 reports how the government policy parameters affect the levels of welfare of different households and aggregate welfare. Several findings emerge from our simulation results. First, higher tax rates of both high-skilled and low-skilled labor raise the aggregate welfare. The intuition is straightforward. There are two effects of raising the taxes on household welfare. The first effect is that a higher tax burden lowers disposal income, and thereby worsens initial consumption and welfare. The second

Table 4

Changes in τ_H, τ_L, θ_G, and welfare effect

τ_H	τ_L	W	W_H	W_L	C_0^H	C_0^L
				$\theta_G = 0.2$		
0.3	0.1	**-84.6789***	**-35.2071**	**-49.4718***	**0.0949***	**0.0579***
0.2	0.1	-88.6835	-37.0364	-51.6471	0.0947	0.0557
0.4	0.1	-81.8551	-34.0042	-47.8509	0.0937	0.0596
0.5	0.1	-79.6865	-33.2025	-46.4840	0.0910	0.0611
				$\theta_G = 0.1$		
0.3	0.1	-82.2077	-33.5781	-48.6296	0.0992	0.0587
0.2	0.1	-78.9632	-31.9897	-46.9735	0.0992	0.0606
0.4	0.1	-76.3026	-30.7194	-45.5832	0.0979	0.0622
0.5	0.1	-86.2559	-35.5825	-50.6734	0.0980	0.0567
τ_H	τ_L			$\theta_G = 0.2$		
0.3	0.1	**-84.6789***	**-35.2071**	**-49.4718***	**0.0949***	**0.0579***
0.3	0.05	-85.2361	-36.0556	-49.1805	0.0928	0.0596
0.3	0.2	-83.9104	-33.6306	-50.2798	0.0991	0.0541
0.3	0.3	-83.2911	-31.9987	-51.2924	0.1037	0.0502
				$\theta_G = 0.1$		
0.3	0.1	-82.2101	-33.5797	-48.6304	0.0992	0.0587
0.3	0.05	-82.8280	-34.4954	-48.3326	0.0967	0.0605
0.3	0.2	-81.3417	-31.8815	-49.4602	0.1040	0.0549
0.3	0.3	-80.9187	-30.3437	-50.5750	0.1088	0.0509

*: the benchmark result

effect is that, as we have shown previously, higher taxes boost economic growth, which is beneficial to welfare. In our model, the second effect outweighs the first one, and thus raising the tax rates improve welfare. In addition, we also see that a higher fraction of infrastructure expenditure is welfare-depressing. The reasoning lies in the fact that a higher fraction of infrastructure expenditure crowds out human capital accumulation. The latter has a stronger positive effect on economic growth. Therefore, as the infrastructure expenditure increases, economic growth reduces, which ultimately deteriorates welfare.

4. Conclusions

This paper constructs an endogenous growth model with human capital, wage inequality, and productive infrastructure.[6] In the household sector, we consider two different type of households (high-skilled and low-skilled) to examine how the government policy affects the wage and welfare of each type. In the productive sector, we consider a representative firm which maximizes its profit and behaves competitively, and we assume that the human capital and government infrastructure expenditure can raise the productivity of final-good production. In the education sector, the government hires high-skilled labor to join the training program to accumulate human capital. Finally, the government maintaining a balanced budget collects income taxes from both labors and allocates the tax revenues among developing human capital, infrastructure expenditure, and the wasteful spending.

Our numerical results show that, along the balance growth path, higher labor taxes improve the economic growth. First, a greater degrees of infrastructure expenditure lower the balance growth rate. This is because an increase in infrastructure expenditure occupies a part of resources which may allocate to develop human capital, which therefore lowers the growth rate of human capital. Second, the balance growth rate rises as the tax rate rise. The economic intuition behind this result is that, given the proportion of infrastructure expenditure hold and the assumption of balanced budget, the

[6] In this model, the proportion of skilled household on final good sector is endogenously determined by government policy.

government will allocate the part of extra revenues on developing human capital when the tax rate increased. However, we also find that policy boosting the growth rate will inevitably lead wage inequality to deteriorate. This means that our results suggest a trade-off between economic growth and wage inequality.

Finally, we explore how the government policy affects social welfare and we find that a greater tax rate of both high-skilled and low-skilled labor raise individual and the aggregate welfare, despite that it worsens wage inequality. In addition, the numerical result shows that a higher fraction of infrastructure expenditure may reduce economic growth and welfare.

Appendix

- Appendix A

Given:

$$\int u\,dv = dv - \int v\,du \tag{A1}$$

Let $u = t,\ dv = e^{-\rho t}dt$ we can obtain :

$$du = dt \tag{A2a}$$

$$v = -\frac{1}{\rho}e^{-\rho t} \tag{A2b}$$

Inserting Eqs. (A2a) and (A2b) into Eq. (A1) we can obtain :

$$\int udv = \int te^{-\rho t}dt$$

$$= t(-\frac{1}{\rho})e^{-\rho t} - \int(-\frac{1}{\rho})e^{-\rho t}dt$$

$$= -\frac{t}{\rho}e^{-\rho t} - \frac{1}{\rho^2}e^{-\rho t} \tag{A3}$$

The welfare function is given by:

$$W_i = \int_0^\infty [lnC_0^i + \gamma_c t + \varphi_i w_p]\, e^{-\rho t}dt\,, i = H, L \tag{A4}$$

Inserting Eq. (A3) into Eq. (A4) we can obtain Eqs. (26) and (28) as :

$$W_i = \int_0^\infty [lnC_0^i + \gamma_c t + \varphi_i w_p]\, e^{-\rho t}dt\,, i = H, L$$

$$= \int_0^\infty lnC_0^i\, e^{-\rho t}dt + \int_0^\infty \gamma te^{-\rho t}dt + \int_0^\infty \varphi_i w_p\, e^{-\rho t}\,dt$$

$$= lnC_0^i(-\frac{1}{\rho}e^{-\rho t})\Big|_0^\infty + \gamma(-\frac{t}{\rho}e^{-\rho t} - \frac{1}{\rho^2}e^{-\rho t})\Big|_0^\infty + \varphi_i w_p(-\frac{1}{\rho}e^{-\rho t})\Big|_0^\infty$$

$$W_i = \frac{1}{\rho}(lnC_0^i + \varphi_i w_p) + \frac{1}{\rho^2}\gamma_c \tag{26 and 28}$$

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