行政院國家科學委員會補助專題研究計畫 ■ 成果報告
□ 期中進度報告

低維系統量子相變之研究

計畫類別：■ 個別型計畫　□ 整合型計畫
計畫編號：96-2112-M-029-006-MY3
執行期間：中華民國96年08月01日至99年07月31日

計畫主持人：栗育文
共同主持人：無
計畫參與人員：無

成果報告類型（依經費核定清單繳交）：□ 精簡報告　■ 完整報告

本報告包括以下應繳交之附件:
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

處理方式: 除產學合作研究計畫、提升產業技術及人才培育研究計畫、
列管計畫及下列情形者外, 得立即公開查詢
□涉及專利或其他智慧財產權,□一年□二年後可公開查詢

執行單位：東海大學物理系

中華民國　99　年　10　月　31　日

低維系統量子相變之研究
Studies on low dimensional quantum phase transitions

計畫編號： 96-2112-M-029-006-MY3
執行期間： 中華民國96年08月01日至99年07月31日
主持人： 栗育文　執行機構及單位名稱: 東海大學物理系
電子信箱 ⟨E-mail⟩： ywlee@thu.edu.tw

一、中文摘要

在本計畫中, 我們主要完成了三件事: 第一, 我們研究一個一維晶格上的玻色子, 在填充數約爲一時, 系統的相與相變。與傳統的流體力學描述的不同處在於, 我們發向必須要引入兩組玻色場才能夠構造一個完整的有效場論, 去描述這格系統的低能行爲。我們發現在此有效場論的架構下, 我們的理論確實給出兩種不同形式的莫特絕緣體。除此以外, 低能有效理論還幫助預測了不同相之間相轉變的普適類。第二, 我們研究一個一維晶格上的玻色子雙鍊模型, 在填充數約爲半滿時, 系統的相與相變。我們利用流體力學的方法推倒出系統的有效場論, 用以描述這個系統的低能行爲。我們發現在此有效場論的架構下, 我們的理論確實給出兩種不同形式的莫特絕緣體。除此以外, 低能有效理論還幫助預測了不同相之間相轉變的普適類 第三, 我們研究了一個具有三體強排斥力的晶格波色系統的超流與莫特絕緣體之相變。此系統除一般的超流相外, 上有一配對超流態。我們利用量子場論中的有效位能方法, 在強耦合展開的架構之下, 成功的推演了該系統序參量之量子作用量。並以此爲依據, 討論了超流到莫特絕緣體之相轉變邊界與相轉變之普適類。

關鍵詞 : 有效理論, 量子相變。

Abstract

In this research project, we accomplish three major things: first, A low-energy effective theory for interacting bosons on a one-dimensional lattice at and near integer fillings is proposed. It is found that two sets of bosonic phase fields are necessary in order to explain the complete phase diagram. Using the present effective theory, the nature of the quantum phase transitions among various phases can be identified. Moreover, the general condition for the appearance of the recently proposed Pfaffian-like state can be realized from our effective action.

Second, We study the phase diagram of two weakly coupled one-dimensional dipolar boson chains at half-odd-integer fillings. We find that the system contains a rich phase diagram. Four different phases are found. They are the Mott insulators, the single-particle resonant superfluid, the paired superfluid, and the bond- or inter-chain density waves. Moreover, the Mott insulating phase can be further classified according to a hidden string order parameter, which is analogous to the one investigated recently in the one-dimensional boson Mott insulator at integer fillings.

Third, we studied the phases and phase transitions of a lattice boson system with three-body hard-core constraint. In addition to the usual superfluid and Mott phases, this system contains an extra paired superfluid phase. Using the effective potential methods borred from quantum field theory, we successfully derived the quantum Ginzburg-Landau functional for the order parameter fields of the system within the framework of strong coupling expansion. Based on this result, we predict the phase boundaries and the nature of the phase transitions in this system.

Keywords: effective theory, quantum phase transitions.

二、緣由與目的

Much effort has been devoted to understanding the effects of competing interactions on quasi-one-dimensional systems. Recent advances in loading ultracold bosonic atoms into an optical lattice lead to the realization of one-dimensional (1D) lattice boson systems, and inspire the investigation on many-body quantum phenomena therein. [1, 2, 3, 4] Besides the nearest-neighbor hopping t and the tunable on-site interaction U, a sizable nearest-neighbor interaction V is now within experimental reach by using the dipolar interaction among atoms. [5, 6] These experiments raise the interest in creating and detecting exotic quantum phases in 1D interacting lattice boson systems.

For 1D lattice bosons at integer filling, the phase diagram obtained by the mean-field calculation includes three different phases. They are the Mott insulator (MI) for large U, a charge density wave (CDW) for large V, and a superfluid (SF) for large t. [7, 8] However, by using the Density Matrix Renormalization Group method, it is found recently that, at filling of $\bar{n} = 1$, there exists a novel insulating phase lying between the CDW and MI phases in the weak coupling region. [9] The phase transition between this new phase and the conventional insulating phases (i.e., the CDW and MI phases) is found to be of second order. Similar to the Haldane phase of quantum spin-one chains, [10] this new insulating phase can be identified by a highly nonlocal string order parameter, and thus called as the Haldane insulator (HI) phase. [9] Another investigation employing quantum Monte Carlo simulations shows that, when $t \ll U \lesssim 2V$, there exists a supersolid (SS) phase lying between the $\bar{n} = 1$ CDW and the $\bar{n} = 1/2$ CDW phases. [11] The CDW-SS transition is found to be of second order with dynamic critical exponent $z = 2$. Some of these new phases are untoniced until very recently.

With an eye on these new developments, it is desirable to have a unified understanding to the rich phases and the quantum phase transitions among them through a suitable low-energy effective theory. In particular, the traditional one-component hydrodynamic theory is clearly unable to describe the above new Mott state and the possible new one-dimensional super-solid state. Our aim in this project is to develop a suitable modified effective theory to gain a better understanding of the above phenomena. The above consistute the motivation behind our first year's research work.

Among many research works devoted to the understanding of the effects of long-range dipolar interactions, we mention the recent work by Emanuele G. Dalla *et al*, [9] who studied the one-dimensional boson insulators within the context of an extended boson Hubbard model (EBHM) by employing the density matrix renormalization group (DMRG) method. By tuning the ratio of the on-site interaction over the hopping amplitude U/t and the ratio of the longer range interaction over the hopping amplitude V/t, the mean field analysis shows that three different *conventional* phases can be reached. These include the Mott insulator at large U, the density wave state for large V and a superfluid state for large t. The surprising thing is that a new intermediate insulating state, the Haldane insulator, which separates itself from the other two insulating states by second order quantum phase transitions was found in Ref. [9]. Moreover, it was shown that such a state possesses a non-vanishing non-local string order, similar to the Haldane phase of the quantum spin-one chain. Such a state is definitely beyond the reach of the traditional one-dimensional hydrodynamic effective theory for the one-dimensional boson superfluid-to-Mott transition. [12]

Knowing the above results, it is desirable to see if similar exotic insulating states can be found in other one-dimensional or quasi-one-dimensional systems. Coupled boson-chain systems have already been discussed in a number of previous publications [13, 14, 15], and the related systems also attracted some renewed interests within the context of cold atoms in both one [16] and higher dimensions. [17]. As far as we know, no exotic Mott insulating phase as we mentioned above has been noticed in these works. In this work, we consider a dipolar boson system of two weakly coupled chains within the framework of the EBHM. Interestingly, we found that the competition of the inter-chain hopping and the inter-chain interaction does lead to two different types of Mott insulating states, with one of them possessing a nontrivial string order. In addition to that, we also found that the inter-chain attraction can give rise to an interesting *paired* superfluid state where the inter-chain bound boson pairs show an algebraic long range superfluid order while the single-boson superfluid correlations decay exponentially. This is the basic ideas begind the second year's research work.

Finally, it was recently suggested that intriguing quantum critical behaviors can occur in attractive bosonic lattice gases with three-body on-site constraint [19, 20]. The system is described by the Bose-Hubbard model with a three-body constraint. The on-site constraint can arise naturally due to large three-body loss processes [21, 22], and it stabilizes the attractive bosonic system against collapse. Therefore, besides the conventional atomic superfluid state (ASF) with non-vanishing order parameters $\langle a \rangle \neq 0$ and $\langle a^2 \rangle \neq 0$ appearing in the weakly-interacting limit, a dimer superfluid phase (DSF) with vanishing atomic order parameter ($\langle a \rangle = 0$) but nonzero pairing correlation ($\langle a^2 \rangle \neq 0$) can be realized for sufficiently strong attraction [21]. It was shown in Refs. [19, 20] that this model provides a simple realization of the physics of Ising quantum transition together with the Coleman-Weinberg mechanism [23] without resorting to the Feshbach-resonant mechanism [24, 25, 26]. While the nature around the ASF-DSF transition has been discussed, the detailed physics of the Mott insulator (MI) to superfluid (either ASF or DSF) transitions is not addressed in Refs. [19, 20]. In the present work, we focus our attention on the MI-ASF and the MI-DSF transitions in this three-body constrained attractive Bose lattice gas.

三、結果與討論

To summarize, in first year's work, a low-energy description, which successfully capture the rich phases of the 1D lattice boson model near integer fillings, is constructed. We believe that the present effective theory provides general applications to related problems. For example, parallel approaches can be used to study models of coupled 1D hard-core lattice bosons, where similar effective theories can be constructed. When local density fluctuations become stronger, more local states with different particle numbers should be kept. Following the above reasoning, an effective theory with more bosonic phase fields should be reached. It is interesting to see if more novel phases can be found in this case.

In the second year's work, we analyze the possible phases of a two-chain boson Hubbard model with long-range interactions. The results of the strong coupling analysis, when one of the inter-chain coupling is dominant, is consistent with the results we gained in the weak-coupling bosonization analysis. Since one of the most important conclusion that follows from the bosonization study is that the interplay between the inter-chain hopping and inter-chain interaction results in two exotic phases — the paired superfluid state and the Mott insulating state with a string order, it is tempting to speculate that the major results of the bosonization analysis in this paper should be valid even when the inter-chain coupling strength is not so weak and can be extended to larger regions in the whole phase diagram.

Using the analytic results obtained in this paper as a guide, we hope that it is helpful for future numerical works to determine the exact phase boundaries of the two-chain EBHM, and more importantly, the exotic insulating phase and the paired superfluid phase can be observed in the future experiments.

Finally, in the last year's work, we have two major results. In the first part, we use the renormalization group method analyze the Feshbach-resonant interacting boson near its unitary limit. Within the framework of epsilin expansion, we found that the systemis always unstable near its unitary limit. In particular, in the molecular side, the system tends to collapses. However, in the atomic side, the the system can exist as a meta-stable state, and we successfully analysize certain thermodynamic properties of the atomic superfluid state including its density distribution and its sound velocity.

In the second part, we analyze the Mott-to-superluid transition of a lattice boson system with three-body hard-core constraint. Due to the constraint, the analytic study of the system becomes notoriously difficult. Another nontrivial part of the system is that when the bosons have attractive two-body interactions, it was predicted that the system can sustain a exoic paired superfluid state — a state where the single particle condensate vanishes while the pair condensate is non-zero. This also increases the difficulty of the theoretical analysis of the system since a straightforward mean-field decoupling is impossible due to the absence of the pairing tunneling term in the original model. Our anaytic approach based on the quantum effective action resolve these two problems simultaneously. It naturally take the constraint into account, and the nontrivial order parameter field does not pose any serious problem to this method. We therefore successfully obtained the phase diagram of the system. Besides, based on the functional form of the Ginzburg-Landau action, we predicted that there is a window of first order phase transition in the phase diagram. In particular, it is found that the DSF phase exists only in a narrow region of chemical potential $\mu/|U|$ for small hopping parameters $t/|U|$. Therefore, carefully tuning system parameters into the suggested parameter regime are necessary to uncover experimentally this novel phase in real ultracold Bose gas in optical lattices.

四、計畫結果自評

茲將本次三年計畫之研究工作的主要成果敘述如下:

1. 在第一年的工作中, 我們透過玻色化的方法平均場的分析, 研究了一維晶格中玻色子在低能下的行爲。
2. 研究主要有兩個重要的結論:
   (a) 當次進鄰交互作用扮演中要角色時, 我們可能要引入一組以上的玻色場來構造低能理論。
   (b) 一維的超固體某種意義上來說可視爲 doped Haldane insulator。因此兩者間的相轉變可能是屬於 commensurate-incommensurate transition 的普適類。
   (c) 由上述的研究結果, 豐富的我們對於低溫, 低維度多體系統量子世界的認識。值得進一步研究之工作包括找尋一個更直接簡單的方法推導此有效作用量, 以及研究此爾種莫特絕緣體相轉變高維推廣。
3. 在第二年的工作中, 我們透過玻色化的方法平均場的分析, 研究了一維晶格中玻色子在低能下的行爲。
4. 該項研究主要的結論:
   (a) 具有非平庸内在結構之莫特絕緣體確實存在於許多一維玻色系統。
   (b) 值得一提的是, 最近有工作表明, 一維的 string order 也許並非標示這類拓樸續的方法[18]。我在下一年度的計畫中, 希望能就這點再進一步研究。
5. 在第三年度的計畫中, 我們主要完成了兩件事, 包含 Feshbach resonant boson system 在 unitary point 之穩定性與可能存在之普適性的分析。此外我們利用一個量子場論的方法分析了具有 constraint 的 lattice boson system 之 Mott transition。相較於前人的方法, 我們的討論顯得簡單明瞭, 同時澄清了 Mott-transition 的特性。這個方法還可以用於分系其他類似系統和 paired superfluid 相關之相轉變。我將在下一次的計畫中, 持續這個方向上的研究。

本次三年度計畫之研究工作, 已發表於 [27] 中的四篇文章。

參考文獻

[1] B. Paredes, A. Widera, V. Murg, O. Mandel, S. Fölling, I. Cirac, G. V. Shlyapnikov, T. W. Hänsch, and I. Bloch, Nature (London) **429**, 277 (2004).

[2] T. Kinoshita, T. Wenger, and D. S. Weiss, Science **305**, 1125 (2004).

[3] T. Stöferle, H. Moritz, C. Schori, M. Köhl, and T. Esslinger, Phys. Rev. Lett. **92**, 130403 (2004).

[4] C. D. Fertig, K. M. O'Hara, J. H. Huckans, S. L. Rolston, W. D. Phillips, and J. V. Porto, Phys. Rev. Lett. **94**, 120403 (2005).

[5] A. Griesmaier, J. Werner, S. Hensler, J. Stuhler, and T. Pfau, Phys. Rev. Lett. **94**, 160401 (2005); A. Griesmaier, J. Stuhler, T. Koch, M. Fattori, Ti. Pfau, and S. Giovanazzi, *ibid.* **97**, 250402 (2006).

[6] *Special issue on Ultracold Polar Molecules: Formation and Collisions*, Eur. Phys. J. D. **31** (2004).

[7] K. Goral, L. Santos, and M. Lewenstein, Phys. Rev. Lett. **88**, 170406 (2002).

[8] D. L. Kovrizhin, G. V. Pai, and S. Sinha, Europhys. Lett. **72**, 162 (2005).

[9] E. G. Dalla Torre, E. Berg, and E. Altman, Phys. Rev. Lett. **97**, 260401 (2006).

[10] F. D. M. Haldane, Phys. Lett. **93**A, 464 (1983); Phys. Rev. Lett. **50**, 1153 (1983).

[11] G. G. Batrouni, F. Hébert, and R. T. Scalettar, Phys. Rev. Lett. **97**, 087209 (2006).

[12] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, New York, 2004).

[13] E. Orignac and T. Giamarchi, Phys. Rev. B **57**, 5812 (1998).

[14] E. Orignac and T. Giamarchi, Phys. Rev. B **57**, 11713 (1998).

[15] P. Donohue and T. Giamarchi, Phys. Rev. B **63**, 180508 (2001)

[16] V. Gritsev, A. Polkovnikov, E. Demler, Phys. Rev. B **75**, 174511 (2007)

[17] Ehud Altman, Walter Hofstetter, Eugene Demler, Mikhail D. Lukin, New J. Phys. **5**, 113 (2003).

[18] Zheng-Cheng Gu, Xiao-Gang Wen, arXiv:0903.1069.

[19] S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **104**, 165301 (2010).

[20] S. Diehl, M. Baranov, A. J. Daley, and P. Zoller, arXiv:0912.3192, and 0912.3196.

[21] A. J. Daley, J. Taylor, S. Diehl, M. Baranov, and P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009).

[22] M. Roncaglia, M. Rizzi, and J. I. Cirac, Phys. Rev. Lett. **104**, 096803 (2010).

[23] S. Coleman and E. Weinberg, Phys. Rev. D **7** 1888 (1973); B. I. Halperin, T. C. Lubensky, and S.-K. Ma, Phys. Rev. Lett. **32**, 292 (1974).

[24] L. Radzihovsky, J. I. Park, and P. B. Weichman, Phys. Rev. Lett. **92**, 160402 (2004).

[25] M. Romans, H. Stoof, and S. Sachdev, Phys. Rev. Lett. **93**, 020405 (2004).

[26] K. Sengupta and N. Dupuis, Europhys. Lett. **70**, 586 (2005); L. Radzihovsky, P. B. Weichman, and J. I. Park, Ann. Phys. **323**, 2376 (2008).

[27] Yu-Wen Lee, Yu-Li Lee, Min-Fong Yang, Phys. Rev. B 76, 075117 (2007), Yu-Wen Lee, Phys. Rev. B 77, 064514 (2008), Yu-Wen Lee and Ming-Fon Yang, Phys. Rev. A 81, 061604 (2010). Yu-Li Lee and Yu-Wen Lee, Phys. Rev. A 81, 063613 (2010).