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碩士論文

房貸保險保費與景氣循環

Mortgage Insurance Premiums and the Business Cycle

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中文摘要

本文首先重新檢視了 Bardhan, Karapandža, and Urošević（2006）（BKU (2006))提出的房貸保險契約評價公式，並重新定義房貸借款者違約時，保險公司損失函數的計算。此外，房價容易被觀察到具有景氣循環的特性。因此，本文進一步根據 Duan, Popova, and Ritchken (2002)提出的馬可夫狀態轉換選擇權評價模型，建構具有景氣循環效果的房貸保險契約的評價模型並提出公式解。根據實證分析結果，狀態轉換模型在美國市場具有良好判斷景氣循環的能力。我們亦發現，在考慮房價具有景氣循環特性的馬可夫轉換模型下計算出來的 2010 年第四季保費高於未考慮此特性所計算的保費。因此，將房價的景氣循環特性納入房貸保險契約的評價應有助於改善房貸保險公司近三年常見的虧損情形。

關鍵字：房貸保險、馬可夫狀態轉換、選擇權訂價、違約風險

JEL classification: G13

Mortgage insurance premiums and the business cycle

Abstract

This research first refines the pricing formula for mortgage insurance (MI) contracts proposed in Bardhan, Karapandža, and Urošević (2006) (BKU (2006)) by re-identifying the setting of conditional losses to insurers. Since the business-cycle property is well observed in housing prices, this research further develops a closed-form solution for valuing MI contracts with business-cycle effects based on the regime-switching option pricing model proposed in Duan, Popova, and Ritchken (2002). Results of real-time analysis show that the ability of the regime-switching model in identifying the business cycle underlying home prices is superior in the U.S. market, and the fair premium of MI contracts with the business-cycle property is higher than that without the property in Quarter 4, 2010. It indicates that incorporating the regime-switching property in valuing MI contracts may facilitate to reduce the losses of insurance companies which are commonly observed in the last three years.

Key words: Mortgage insurance; Markov regime-switching; Option pricing; Default risk

JEL classification: G13

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1 Introduction

Mortgage insurance (MI) contracts provide an important tool for lenders to hedge their risk exposures by insuring mortgages against default. The common concern of lenders, borrowers and insurers concerning MI contracts is the fair premium of the insurance contracts.

The critical problem for valuing a MI contract centers on the determination of mortgage termination, which may be caused by either prepayment or default on the underlying loan. Generally speaking, prepayment decisions are in relation to the term structure of interest rate, while default decisions depend on the housing price. In the literature, Kau, Keenan, Muller, and Epperson (1992, 1995), Kau, Keenan, and Muller (1993), and Kau and Keenan (1995, 1999) value mortgage loans and MI contracts by using dynamic programming methods or backward pricing models. Nevertheless, all of these methods involve complex numerical procedures. As pointed out by Bardhan, Karapandža, and Urošević (2006) (BKU (2006), hereafter), the complexity inherent in these numerical approaches may not be warranted in the consideration of fitting the model to the data.

Instead of developing numerical approaches, BKU (2006) show that the payoff from MI contracts when default occurs is equivalent to the payoff from a bear spread created by put options and thus develop an option-pricing framework to price MI contracts in closed form. The pricing formula proposed in BKU (2006) not only allows the realized loss for the insurer in case of the borrower's default occurring at time t to depend on the time t collateral price and loan balance, but also permits legal inefficiencies, which may lead to a delay in the repossession of loan collateral, to be taken into account in valuing MI contracts. As the first research proposing the closed-form formula in valuing MI contracts, the most important advantage inherent in BKU approach is ease of use.

Since the critical idea used in BKU (2006) is to regard the payoff from MI contracts as a bear spread, we thus name the method that prices MI contracts through a bear spread as the BKU approach.

In practice, the insurer of MI contracts bears the loan balance at time t and has the right to possess the underlying collateral given that the borrower defaults at time t. The conditional losses from MI contracts is usually with a maximum limit. Accordingly, the conditional losses for the insurer should be the difference between the time t loan balance and collateral price, if the maximum loss is not hit. This is also the content of MI contracts investigated in Kau, Keenan, Muller, and Epperson (1995) (KKME (1995), hereafter). However, we observe that the MI contract defined in BKU (2006) is different to that KKME (1995). In contrast with the use of time t value for the loan balance, the loss for the insurer defined in the BKU contract is the difference, with a maximum limit, between the time $t-1$ value of the loan balance and the time t housing value. Since the KKME contract is more in line with the reality, this research revalues MI contracts by refining the BKU's formula based on the KKME contract. This is the first contribution in this research. One distinguish feature of this proposed formula is the dynamics of housing prices being assumed to follow the geometric Brownian motion (GBM), this proposed model is also named as the GBM model in the following.

It is well known that the value of derivatives depends on the dynamics of the underlying asset. Similarly, the premium of MI contracts depends on the process of housing prices. A number of empirical studies have provided evidences concerning the linkup between housing prices and the business cycle. To illustrate, Iacoviello (2005) proposes VAR evidence on housing prices and the business cycle. Davis and Heathcote (2005) show that housing prices are cyclical. Clark and Coggin (2009) fit a classic smooth trend plus cycle structural time series model to U.S. regional house price. Edelstein and Tsang

(2007) develop a housing cycle model for residential housing market cyclical dynamics. Moreover, Jin and Zeng (2004) explain the correlation between residential investment, house prices, and the business cycle properties by developing a three-sector quantitative dynamic stochastic general equilibrium model. Although the BKU approach is very easy to be applied, the key assumption that the dynamics of house prices follow the geometric Brownian motion is far from to capture the business-cycle property behind housing prices at all. It motives us to fill the gap by incorporating business-cycle property back housing prices into the valuation of MI contracts. Since data with the cyclical feature are found to be well described by Markov regime-switching (RS) model, this research develops a pricing approach for valuing MI contracts with business-cycle effects based on the regime-switching option pricing model.

Our study is not the only to incorporate regime-switching mechanisms into the valuation of MI contracts. To value MI contracts under business cycles, Lin and Chuang (2010) incorporate the regime-switching mechanism into the BKU (2006) methodology and develop a semi-closed-form approach to value options under regime switching. The proposed approach is not exactly in closed form since the unconditional probability of state k occurring j times during the life of options, which is one of terms in their pricing formula, is calculated by a backward device in Lin and Chuang (2010). Unlike Lin and Chuang (2010), this research investigates the premium of MI contracts with the business-cycle property based on the KKME contract. Herein, the business-cycle characteristic is modeled by the regime-switching option pricing approach proposed in Duan, Popova, and Ritchken (2002) (DPR (2002), hereafter), which not only allows asset innovations to have feedback effects on volatilities, but also permits regime shift risk to be priced. Although the closed-form formula for pricing MI contracts proposed in this research is based on a two-state uni-directional regime-switching model, the MI premium under a

N-state bi-directional regime-switching framework is able to be calculated under DPR's (2002) framework as well.

The remaining parts of this paper are arranged as follows. Section 2 reviews the literature concerning MI contracts and the business cycle property of housing prices. Section 3 introduces the theoretical framework of the BKU approach for valuing MI contracts and refines the MI pricing formula based on the KKME contract. We also conduct a numerical analysis to compare the BKU and KKME premiums. Section 4 develops the pricing formula of MI contracts by taking the business-cycle property underlying housing prices into account. We also investigate sensitivity of premiums from the RS model to transition probabilities in this section. Section 5 empirically estimates the parameters of the GBM model and RS model and calculates the MI premiums in Quarter 4, 2010 for the U.S. market, respectively. We find that the ability of the RS model in capturing the business-cycle characteristic is satisfactory. Moreover, the fair premium of MI contracts is higher than that of the GBM model in Quarter 4, 2010 when we take the business-cycles property behind housing prices into account. Concluding remarks are given in the last section.

2 Literature review

2.1 Mortgage insurance

As mention in Kau, Keenan, and Muller (1993), Kau and Keenan (1995) and Kau, Keenan, Muller, and Epperson (1995), a MI contract is an asset to guard against the mortgage's default since its payoff is determined by the mortgage contract. Thus, the MI contract can be regarded as the mortgage-backed security. This is also the reason why assumptions and approaches for valuing MI contracts usually follow literature about mortgage contracts. Accordingly, the literature reviewed in the following includes documents concerning mortgage contracts and MI contracts.

Kau and Keenan (1995) survey theoretical work for pricing the mortgage and mention that the option-pricing approaches, or namely contingent claims models, have been employed to price mortgage contracts or MI contracts. As pointed out in Kau and Keenan (1995), there are two state variables in the economic environment relative to a mortgage: the price of the asset collateralized in the mortgage and the interest rate. The price of the asset collateralized is usually assumed to follow a log-normal process such as a geometric Brownian motion, like Epperson, Kau, Keenan, and Muller (1992, 1993, 1995), Schwartz and Torous (1992), and Titman and Torous (1989). The assumption, that the interest rate follows a mean-reverting process, is the most popular way in the literature, including Schwartz and Torous (1992), Titman and Torous (1989), and Epperson, Kau, Keenan, and Muller (1992, 1993, 1995). Nevertheless, Schwartz and Torous (1989) provide a two-state process with the spot rate and the long rate.

They also classify mortgage contracts into different categories, including fixed-rate mortgages, adjustable-rate mortgages, graduated payment mortgages, and price level adjusted mortgages.

Along this strand of literature, the common method used in valuing mortgage contracts is to derive the partial different equation (hereafter, PDE) that involves the collateral price and interest rate and then adopt numerical methods e.g., forward pricing and backward pricing methods, to obtain the solution of the PDE. For example, Titman and Torous (1989) use a contingent-claims model for pricing commercial mortgages. The model assumes that the valuation of a mortgage can be summarized by two state variables. One is the risk-free interest rate, which is assumed to follow a mean-reverting square root diffusion process. The other is the value of the mortgaged building, which is assumed to follow a log-normal diffusion process. The two state variables are correlated at the same time. Titman and Torous (1989) derive the PDE with appropriate boundary and initial conditions and solve this PDE numerically.

Mortgage insurances are the contracts for transferring the contingent loss of the lender in the mortgage contract to the insurance. Under this concept, Kau, Keenan, and Muller (1993) use an option-pricing approach to investigate the private mortgage insurance and find how changes in the mortgage contract or changes in the economic environment influence the MI values. They find that impacts on insurance prices induced from the loan-to-value ratio are less than those recognized in theory.

Kau, Keenan, Muller, and Epperson (1995) establish a model to price fixed-rate mortgages and mortgage insurances with the borrower's decisions, i.e., default and prepayment. In their framework, the value of the house and the interest rate are assumed to follow the stochastic processes, which are equivalent to Titman and Torous (1989), Kau, Keenan, and Muller (1993), and Kau, Keenan, Muller, and Epperson (1992, 1995). They also demonstrate that the PDE for valuing mortgages and MI contracts is solely a function of the house price and the interest rate, in which the two stochastic processes are correlated. They use a series of numerical procedures to analyze the different features of the mortgage contract under a variety of economic conditions, including the changes in interest rate, house price variance, mortgage contract rate, and upfront points charged. The numerical results show that the value of insurance increases with the volatility of housing prices and the loan-to-value ratio, but decreases as the volatility of the interest rate grows.

We also note that the boundary conditions are related to the default and prepayment decisions in these studies, and thus these decisions influence the valuation of MI contracts as well. It implies that the probabilities of default is endogenous. Although these researches allow the default probability to be endogenous, no closed-form solution of MI contracts exist under this framework. Indeed, the numerical methods used in the strand of literature are complex and difficult to be applied.

To simply the numerical procedures, Dennis, Kuo, and Yang (1997) develop a framework to price mortgage insurance premiums in the light of exogenous termination probabilities. According to the general terminal condition of the mortgage insurance given by KKME (1995) and the mortgage insurance premium structure proposed in Dennis, Kuo, and Yang (1997), BKU (2006) first show that mortgage insurance payoff diagram is a portfolio of two European put options when the borrower's default occurs at time t. The portfolio is composed of a long position and a short position in European put options with different strike prices. Unlike the two state variables used in the PDE approach, BKU (2006) simplify the state variable to be one, i.e., the house value. Additionally, they assume that the probabilities of borrower default is exogenously determined at each period. They thus are able to develop a new option-based method in closed form for valuing the MI contracts instead of the numerical procedures. The premium of the MI is calculated as the present value of the accumulated expected loss weighted by the exogenously default probabilities. They also estimate the unconditional default probabilities for the case of Serbia based on the proposed formula. Furthermore, they suggest that the cost of legal inefficiency significantly affects the MI premiums.

One common assumption used in the literature is to assume the dynamic of housing prices to be the geometric Brownian motion. However, the dynamic process is unable to capture various features of housing prices in reality at all. Accordingly, some studies notice that catastrophic events deeply influence the changes in housing prices such as Kau and Keenan (1996) and Chen, Chang, Lin, and Shyu (2010) and incorporate a jump diffusion form into the process of housing prices. For instance, Kau and Keenan (1996) consider the impact of catastrophic events on housing price process and demonstrate a jump diffusion form with a Poisson distribution. Specifically, they employ a numerical method for pricing mortgage insurance. Chen, Chang, Lin, and Shyu (2010) assume that the housing price process follows the jump diffusion process and provide the closed form solution for the valuation of MI based on the framework of BKU (2006). The impacts of the volatility of housing prices on premiums is consistent with those in the literature. The impact of jump risk on premiums is positively related to abnormal volatility of jump size and shock frequency of the abnormal event, but negatively related to the mean of jump size.

Business cycle is another property that can be observed from housing market usually. In order to capture the business cycle feature in housing prices, Lin and Chuang (2010) first incorporate this feature into the valuation of MI contracts. Specifically, they assume that the housing price process follows the Markov-switching model and derive a semi-closed-form formula for pricing MI contracts based on the BKU (2006) contract. They employ a backward method to calculate the unconditional probability of state k occurring j times during the life of options, which is one of terms in their MI pricing formula. They also adopt a series of tests to identify the properties of the housing price such as business cycle, volatility clustering, asymmetric returns, and leptokurtic. Numerical results show that both the volatilities in different states are positively related to premiums in otherwise identical. They also find that the number of periods in high level volatility state is positively related to premiums, whereas the number of periods in low level volatility state is negatively related to premiums.

2.2 Housing prices and the business cycle

A number of literature have provided evidences concerning that housing prices display a special characteristics over the business cycle and have correlations with some economic variables. To illustrate, Jin and Zeng (2004) explore correlation between residential investment and house prices. They also develop a three-sector quantitative dynamic stochastic general equilibrium model to explain the business cycle properties concerning residential investment and house prices.

Iacoviello (2005) further shows the relation between house prices, borrowing constraints, and monetary policy in the business cycle and proposes VAR evidence on housing prices and the business cycle on quarterly data. Davis and Heathcote (2005) build a neoclassical multisector stochastic growth model to understand the dynamics of residential investment. The result of simulating the model economy show that housing prices are cyclical in the United States.

Edelstein and Tsang (2007) develop a housing cycle model for residential housing market cyclical dynamics. Using single-family housing units for the four largest metropolitan statistical areas (M.S.A.) in California: San Francisco, Los Angeles, San Diego and Sacramento, the empirical analysis sorts national-macro, regional and local market variable effects upon cycles.

Finally, Clark and Coggin (2009) use the U.S. quarterly regional house prices compiled and maintained by the U.S. Office of Federal Housing Enterprise Oversight (OFHEO).

They present a statistical analysis of the time series properties of trend, cycles and convergence of U.S. regional house prices and fit a classic smooth trend plus cycle structural time series model to U.S. regional house price.

Based on the findings in the literature, the business-cycle feature is an important property of housing prices. Accordingly, this research follows the idea of Lin and Chuang (2010) to incorporate the business-cycle property into the valuation of MI contracts and intent to develop the closed-form formula for MI contracts based on the regime-switching option pricing model proposed in DPR's (2002) model.

3 Theoretical framework

3.1 The theoretical framework in BKU (2006)

This research investigates the fair premium of MI contracts by incorporating the feature of business cycles inherent in housing prices. Before developing our proposed formula for MI premiums, we introduce the setting of the economic environment used in BKU (2006) in this subsection.

The asset underlying MI contracts is a mortgage loan, in which the borrower provides property to be collateral for a bank loan and promises transferring the collateral to the lender if he cannot redeem the loan. Although the mortgage is secured by collateral, the lender still exposes potential loss due to the possibility of the housing value being less than the loan balance when the borrower cannot afford to pay installments.

Denote V_t as the value of collateral at time t and $\mathcal{R}$ as the annual risk-free rate. At origination, i.e., $t = 0$, the lender issues a T-period mortgage loan with a fixed mortgage rate c and a loan amount of B_0, where $B_0 = L_V V_0$ and L_V is the loan-to-value ratio. Since mortgage loans are usually redeemed by installments, we assume that the borrower pays a installment y back at each time t, where $t \in (0, T]$. Without loss of generality, the time t loan balance, B_t, is equal to the total present value of the unpaid payments ranging from time $t + 1$ to time T, that is:

$$B_t = \frac{y}{c}\left(1 - \frac{1}{(1+c)^{T-t}}\right). \tag{1}$$

As mentioned above, the lender still takes risk when mortgage loans are secured by collateral, especially during the house-market depression. Fortunately, insurance markets allow lenders to transfer this risk through MI contracts. According to BKU (2006), the amount that the insurer of a MI contract compensates the lender in case of default occurring at time t is:

$$Loss_t^{BKU} = \max(0, \min(B_{t-1} - V_t, L_R B_{t-1})), \tag{2}$$

where L_R limits the maximum loss for the insurer and is called the loss ratio. Equation (2) clearly indicates that the insurer compensates the lender for his loss when the borrower cannot afford to pay installments. The amount is the deficiency of the proceeds from the sale of the collateral and loan balance, i.e., $B_{t-1} - V_t$, if any. But the maximum amount paid by the insurer is the ratio L_R of the loan balance, i.e., $L_R B_{t-1}$. Figure 1 plots the payoff from MI contracts displayed in Equation (2). An important contribution of BKU (2006) is to demonstrate that the mortgage insurance payoff diagram displayed in Figure 1 can be regarded as a portfolio of two European put options when the borrower's default occurs at time t. It follows that the valuation of MI contracts can be implemented based on the option pricing theorem.

Assume that the housing price follows the geometric Brownian motion, i.e.,

$$\frac{dV_t}{V_t} = (\mathcal{U} - \mathcal{S})dt + \Sigma dz_t, \tag{3}$$

where $\mathcal{U} - \mathcal{S}$ is the expected annual rate of collateral appreciation, $\mathcal{S}$ denotes the rental yield, z_t represents a standard Wiener process, and $\Sigma > 0$ is the annual volatility coefficient. BKU (2006) develop a closed-form formula for valuing MI contracts based on Equation (2). Denote $Put(K_{i,t}, t)$ as the value of a put option with a strike price $K_{i,t}$ and a maturity date t, BKU (2006) demonstrate that the current value of $Loss_t^{BKU}$, i.e., $\mathcal{L}_t^{BKU}$, can be valued by:

$$\begin{aligned}\mathcal{L}_t^{BKU} &= e^{-\mathcal{R}t} E^Q\{\max(K_{1,t}^{BKU} - V_t, 0) \mid \mathcal{F}_0\} - e^{-\mathcal{R}t} E^Q\{\max(K_{2,t}^{BKU} - V_t, 0) \mid \mathcal{F}_0\} \\ &= Put\left(K_{1,t}^{BKU}, t\right) - Put\left(K_{2,t}^{BKU}, t\right),\end{aligned}$$

where $E^Q\{\cdot \mid \mathcal{F}_0\}$ denotes the expectation conditional on $\mathcal{F}_0$ under measure Q,

$$K_{1,t}^{BKU} = B_{t-1},$$

and

$$K_{2,t}^{BKU} = (1 - L_R)B_{t-1}.$$

Herein, the value of put options, $Put\left(K_{1,t}^{BKU}, t\right)$ and $Put\left(K_{2,t}^{BKU}, t\right)$, can be priced by the Black-Scholes formula. Since the loss for the insurer may happen at any time t, where $t \in (0, T]$, the fair premium of the mortgage insurance contract, FP_0^{BKU}, is given by the following expression:

$$FP_0^{BKU} = \sum_{t=1}^{T} P_d(t)\mathcal{L}_t^{BKU},$$

where $P_d(t)$ is the unconditional probability that the borrower defaults at time t. The critical idea used in BKU (2006) is to regard the payoff from MI contracts as a bear spread, we thus name the method that prices MI contracts through a bear spread as the BKU approach.

3.2 Revaluing MI premiums based on the KKME contract

In practices, the insurer of MI contracts bears the loan balance at time t and has the right to possess the underlying collateral in case of default happening at time t. Similar to what defined in Equation (2), the conditional losses from MI contracts is usually with a maximum limit L_R, which is presented as a percentage of the loan balance. Thus the conditional losses for the insurer should be the difference between the time t loan balance and collateral price, if the maximum loss is not hit. This is also the content of MI contracts investigated in Kau, Keenan, Muller, and Epperson (1995) (KKME (1995), hereafter). However, we observe that the MI contract of BKU (2006) is different from that of KKME (1995) in identifying the value of the loan balance when the borrower defaults.

Specifically, default happening at time t indicates that the borrower is unable to pay both the time t installment payment y and the remaining $T-t$ future installments, which is worth B_t. As shown in Equation (2), the loan balance in case of default occurring at time t used in BKU contract is the time $t-1$ value of these $T-t+1$ future installments, not the time t value of this loan balance. Since the insurer of MI contracts takes all responsibility for debt once the borrower cannot redeem the loan, the KKME contract is more in line with the reality. It motives us to refine the pricing formula proposed in BKU (2006) based on the KKME contract.

According to the KKME contract, the remaining loan balance when default occurs at time t is $B_t + y$, or equivalent to $(1+c)B_{t-1}$, rather than B_{t-1}. It follows that the loss to the insurer in case of default at time t should be refined as:

$$Loss_t^{KKME} = \max(0, \min(B_t + y - V_t, L_R(B_t + y))). \tag{4}$$

Similarly to the result proposed in BKU (2006), Equation (4) indicates that the potential loss borne by the insurer is equivalent to the cash flow from a portfolio of put options, that is:

$$\begin{aligned} Loss_t^{KKME} &= \max(0, \min(B_t + y - V_t, L_R(B_t + y))) \\ &= \max(K_{1,t} - V_t, 0) - \max(K_{2,t} - V_t, 0), \end{aligned} \tag{5}$$

where

$$K_{1,t} = B_t + y,$$

and

$$K_{2,t} = (1 - L_R)(B_t + y).$$

Particularly, the loss for the insurer conditional on default happening at time t equals the cash flow from a portfolio of two put options on the collateral price. Herein, one of the two options is a long position in a European put option with a strike price of $K_{1,t}$ and a maturity of t, and the other is a short position in a European put option with a strike price of $K_{2,t}$ and a maturity of t. Similarly to BKU (2006), the option pricing theorem can be applied to price the MI contract defined in Equation (4) as well.

Based on the option pricing theorem, the current value of $Loss_t^{KKME}$, i.e., $\mathcal{L}_t^{KKME}$, can be calculated by:

$$\begin{aligned}\mathcal{L}_t^{KKME} &\equiv e^{-\mathcal{R}t}E^Q\{\max(K_{1,t}-V_t,0)\mid \mathcal{F}_0\} - e^{-\mathcal{R}t}E^Q\{\max(K_{2,t}-V_t,0)\mid \mathcal{F}_0\} \\ &= Put(K_{1,t},t) - Put(K_{2,t},t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} Put(K_{i,t},t) &= K_{i,t}e^{-\mathcal{R}t}N(-d_2(K_{i,t})) - V_0e^{-\mathcal{S}t}N(-d_1(K_{i,t})), \qquad (7)\\ d_1(K_{i,t}) &= \frac{\ln(V_0/K_{i,t}) + (\mathcal{R}-\mathcal{S}+0.5\Sigma^2)\,t}{\Sigma\sqrt{t}}, \\ d_2(K_{i,t}) &= d_1(K_{i,t}) - \Sigma\sqrt{t}, \qquad \forall\ i=1,2. \end{aligned}$$

Accordingly, the fair premium of the MI contract under the KKME contract, FP_0^{KKME}, is given by the following expression:

$$FP_0^{KKME} = \sum_{t=1}^{T} P_d(t)\mathcal{L}_t^{KKME}. \tag{8}$$

Obviously, the proposed formula for pricing the KKME contract displayed in Equations (6)-(8) is similar to the original formula of BKU (2006) except for the strike prices of put options. Since the geometric Brownian motion is the key assumption for the price dynamics of collateral prices, we also name this proposed pricing formula displayed in Equations (6)-(8) as the GBM model.

Subsection 3.3 will show that the difference between the KKME premium and BKU premium is too obvious to be ignored. As mentioned above, the KKME contract displayed in Equation (4) fits in more with the reality. As a result, we develop the MI pricing formula with the property of the business cycle based on this contract in what following.

3.3 Numerical comparisons between the BKU contract and KKME contract

This subsection conducts numerical analysis to investigate the difference of premiums between the BKU contract, FP_0^{BKU} and KKME contract, FP_0^{KKME}.

To be consistent with the market convention, we report the fair premium on an annual-pay basis and represent this annual premium as a ratio of the underlying mortgage loan, B_0, in the following. This premium ratio is called as Equivalent Annual Premium (EAP). Since an annual premium terminates once the mortgage loan defaults, a reasonable return required by the insurer for transferring the fair MI premium from a lump-sum-payment basis to an annual basis is the mortgage contract rate, c. Accordingly, the way to transfer the fair MI premium to EAP is given by:

$$EAP = \frac{FP_0^i}{B_0} \times \frac{c}{(1+c)(1-\frac{1}{(1+c)^T})}. \tag{9}$$

Here, FP_0^i represents the fair premium calculated from the contract i, that is the BKU or KKME contract.

Table 1 compares the EAP under the KKME contract to that of the BKU contract for various maturities of MI contracts, T, volatility of collateral price, Σ, loan-to-value ratios L_V, mortgage rates c, risk-free rates $\mathcal{R}$, loss ratios L_R, and unconditional default probabilities $P_d(t)$. Common parameters for each of the cases in Table 1 are given by: $T = 30$, $\Sigma = 4\%$, $L_V = 0.9$, $c = 5\%$, $\mathcal{R} = 0.5\%$, $L_R = 0.75$, $P_d(t) = 0.02$, $\mathcal{S} = 5\%$, and $V_0 = 1,000,000$ except when a remark is made.

As expected, Table 1 displays that both the BKU and KKME premiums increase with the time to maturity T, volatility Σ, loan-to-value ratio L_V, mortgage contract rate c, and loss ratio L_R, whereas both of the two premiums decrease as the risk-free rate $\mathcal{R}$ grows. Specifically, increases in the volatility of housing prices and the length of maturity induce more uncertainty about the future housing price. Both cases more possibility that the underlying mortgage defaults during the life of MI contracts. Similarly, the risk exposures of insurers increase when the loan-to-value ratio L_V and loss ratio L_R grow. The former raises the probability of default occurring, and the latter enlarges the possible loss of insurers when default happens. Moreover, a higher mortgage rate c burdens borrowers with a heavier installment payment y and thus amplifies the possibility of borrowers being unable to afford future installments. Those are reasons why EAP increases with the maturity T, volatility Σ, loan-to-value ratio L_V, loss ratio L_R, and mortgage contract rate c. Finally, the phenomenon that EAP decreases as the risk-free rate $\mathcal{R}$ raises is consistent with the option pricing theorem.

Table 1 also shows that the refined MI premium, i.e., the KKME price, is never less than the BKU premium, and the impact of this revision on premiums may be significant. To illustrate, under the setting of $T = 30$ and $P_d(t) = 0.02$, the premiums calculated from the KKME and BKU contract are 94.23 and 86.85 bps, respectively. It follows that the absolute underestimation induced from the BKU contract under this case reaches 7.83%. Moreover, the absolute pricing deviation come from BKU premiums enlarges as the mortgage contract rate c, risk-free rate $\mathcal{R}$, and loss ratio L_R grow, whereas it declines as the maturity T, volatility Σ, and loan-to-value L_V increase. Within examples provided in Table 1, the maximum pricing deviation induced from the BKU price approaches to 19.70% under the case of $T = 20$ and $P_d(t) = 0.005$. Nevertheless, the minimum undervaluation from the BKU contract is still 6.42% in Table 1. It indicates that the difference between the KKME and BKU premium is very significant and cannot be ignored.

To clarify the pricing deviation resulted from the BKU contract more clearly, Figure 2 compares the impacts of various parameters, including the maturity of MI contracts T, volatility of collateral prices Σ, loan-to-value ratio L_V, mortgage contract rate c, risk-free rate $\mathcal{R}$, and loss ratio L_R, on BKU and KKME premiums. Common parameters used in Figure 2 are identical to those in Table 1, except for the parameter displayed in the horizonal axis for each sub-figures. Again, we observe that the BKU price usually undervalues the fair premium. As shown in Figure 2, the absolute underestimation in basis points induced by the BKU contract widens when the maturity T, loan-to-value ratio L_V, mortgage contract rate c, and loss ratio L_R increase. On the contrary, the lower the risk-free rate $\mathcal{R}$ is, the larger underestimation resulted from the BKU contract. The pricing error induced from the BKU contract is also found to be negatively correlated with the value of the collateral volatility Σ. However, the pricing deviation of the BKU contract does not disappear significantly no matter how large the value of the collateral volatility Σ is.

In the last three years, many insurance companies have been reported to suffer from great losses. For example, a CNBC report on October, 2010 pointed out that mortgage insurers have lost big in the last two years on all types of loan. Reports from CNBC, HousingWire, and Bloomberg also indicated that mortgage insurance generally have losses in those years. For example, MGIC Investment Corp. (MTG), the largest U.S. guarantor of home loans, announced that their net loss was $33.7 million on April, 2011, which has been unprofitable in 14 of the past 15 quarters. Mortgage insurer Radian Group (RDN) also announced that losses for 2010 reach $1.8 billion, ballooning from $147.9 million in losses for 2009. By the report on November, 2010, mortgage insurer PMI Group (PMI) lost $281.1 million in the third quarter of 2010 and the company set aside more funds for potential losses. The loss more than tripled the $87.9 million lost in the third quarter of last year. In 2009, Old Republic International Corp. (ORI), which provides residential mortgage insurance as well as title insurance and other real estate transfer-related services, reported a quarterly net loss of $126.5 million, and warns of continued slump into 2010.

We note that undervaluing the premiums may be one of reasons that result in the losses of insurance companies. Since the KKME premium is greater than the BKU price, the refined model for valuing MI contracts may facilitate to reduce the losses of insurance companies which are commonly observed in the last three years.

4 Valuing mortgage insurance contracts with the business-cycle property

4.1 The MI pricing formula under the regime-switching framework

A number of empirical studies have provided evidences about the linkup between housing prices and the business cycle. To illustrate, Iacoviello (2005) proposes VAR evidence on housing prices and the business cycle. Davis and Heathcote (2005) show that housing prices are cyclical. It motives us to fill the gap by incorporating business-cycle property back housing prices into the valuation of MI contracts. Since data with the cyclical property have well known to be described by Markov regime-switching models, we adopt the regime switching option pricing model proposed in DPR (2002) to model the house prices.

The business cycle means that the housing prices have the cyclical property, which is described especially by the different level of volatilities well. It is common to be described as two state, economic recession and expansion. For instance, Hamilton (1989) proposes an approach to modeling changes in regime by analyzing U.S. real GNP. The results show that it is appropriate to date the business cycle by using two state. That is why this paper concentrates on the two-state regime in housing prices.

Let σ_{t+1}^2 be the conditional variance of the logarithmic return at date t that holds for the period $[t, t+1]$ and r as the risk-free rate over the period $[t, t+1]$. According to DPR (2002), a two-state uni-directional regime switching model can be written as:

$$\ln \frac{V_{t+1}}{V_t} = r + \lambda\sigma_{t+1} - s - \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}\varepsilon_{t+1}, \tag{10}$$

$$\sigma_{t+1} = \begin{cases} \delta_1, & \text{if } |\xi_t| < \Phi(\sigma_t), \\ \delta_2, & \text{if } |\xi_t| \geq \Phi(\sigma_t) \end{cases} \tag{11}$$

and

$$\begin{bmatrix} \varepsilon_{t+1} \\ \xi_{t+1} \end{bmatrix} |\mathcal{F}_t \overset{P}{\sim} N(0_{2\times 1}, I_{2\times 2}).$$

Here, λ stands for the market price of risk, δ_k $(k = 1, 2)$ denotes the volatility level, and the conditional volatility σ_{t+1} depends on the values of σ_t and ξ_t. The random variable ξ_t is independent of ε_t. Based on Equation (11), the volatility transition probability matrix is:

$$\mathcal{P} = \begin{bmatrix} N(\Phi(\delta_1)) - N(-\Phi(\delta_1)) & 1 - N(\Phi(\delta_1)) + N(-\Phi(\delta_1)) \\ N(\Phi(\delta_2)) - N(-\Phi(\delta_2)) & 1 - N(\Phi(\delta_2)) + N(-\Phi(\delta_2)) \end{bmatrix}.$$

To simplify notation and make it to be consistent with that in Hamilton (1989), we further denote the transition probability matrix as:

$$\mathcal{P} = \begin{bmatrix} N(\Phi(\delta_1)) - N(-\Phi(\delta_1)) & 1 - N(\Phi(\delta_1)) + N(-\Phi(\delta_1)) \\ N(\Phi(\delta_2)) - N(-\Phi(\delta_2)) & 1 - N(\Phi(\delta_2)) + N(-\Phi(\delta_2)) \end{bmatrix} \equiv \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where p_{ij} represents the probability of switching to regime j from regime i and $\sum_{j=1}^{2} p_{ij} = 1$ for all i. Please note that the values of $\Phi(\delta_1)$ and p_{11} are corresponding one by one and so as the values of $\Phi(\delta_2)$ and p_{22}.

Denote the time interval between $[t, t+1]$ as Δt. For the purpose of comparison, we note that the discrete-time version for Equation (3) can be written as:

$$\ln \frac{V_{t+1}}{V_t} = (\mu - s - \frac{1}{2}\sigma^2) + \sigma\varepsilon_{t+1}, \tag{12}$$

where $\mu \equiv \mathcal{U}\Delta t$, $s \equiv \mathcal{S}\Delta t$, and $\sigma \equiv \Sigma\sqrt{\Delta t}$. Moreover, ε_{t+1} is a standard normal random variable and Δt depends on the data frequency. By assuming the number of regime to be one, Equation (10) is reduced to:

$$\ln \frac{V_{t+1}}{V_t} = r + \lambda\sigma - s - \frac{1}{2}\sigma^2 + \sigma\varepsilon_{t+1}. \tag{10'}$$

Recognize that the drift term of price appreciation equals to the sum of risk-free rate and risk premium, i.e., $\mu = r + \lambda\sigma$, the regime switching model displayed in Equation $(10^{'})$ is equivalent to the discrete-time version of the geometric Brownian motion displayed in Equation (12) given that the number of regime is set to be one.

DPR (2002) provide a closed-form solution for European options when the dynamics of the underlying asset follow the two-state uni-directional regime switching model displayed in Equation (10). Denote $\sigma_{t+1} = \delta_k$ as the regime during the period $[t, t+1]$, i.e., it illustrates the volatility for the period $[t, t+1]$. Let $Put(K_{i,t}, t|\sigma_0 = \delta_k)$ be the time 0 price of a European put option with a strike price $K_{i,t}$, expiration t and initial volatility δ_k, where $k = 1, 2$. The fair value of $Put(K_{i,t}, t|\sigma_0 = \delta_k)$ is given by:

$$Put(K_{i,t}, t|\sigma_0 = \delta_k) = \sum_{j=0}^{t} \gamma_{t,j|\sigma_0=\delta_k} Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j), \tag{13}$$

where

$$Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j) = Ke^{-rt}N(-d_{2j}) - V_0 e^{-st}N(-d_{1j}),$$

and

$$d_{1j} = \frac{\ln(V_0/K) + rt - st + 0.5\theta_j^2}{\theta_j},$$

$$d_{2j} = d_{1j} - \theta_j,$$

$$\theta_j^2 = j\delta_1^2 + (t-j)\delta_2^2, \qquad j = 0, 1, \cdots, t.$$

Here, N denotes the number switching from state k to state 1 during the remaining life of the option, and $Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j)$ represents the time 0 price of the European put option, given that initial volatility is δ_k and the number switching from state k to state 1 during the remaining life of the option is j times. The probability $\gamma_{t,j|\sigma_0=\delta_k}$ represents that in t periods the number of visits from state k to state 1 is j. Based on DPR (2002),

$\gamma_{t,j|\sigma_0=\delta_k}$ is calculated as follows:

$$\gamma_{t,j|\sigma_0=\delta_1} = \begin{cases} p_{12}p_{22}^{t-1}, & \text{for } j=0 \text{ and } t=1,2,\cdots,T \\ p_{11}, & \text{for } j=1 \text{ and } t=1 \\ p_{11}p_{12}p_{22}^{t-2} + (t-2)p_{12}^2 p_{21}p_{22}^{t-3} + p_{12}p_{21}p_{22}^{t-2}, & \text{for } j=1 \text{ and } t=2,3,\cdots,T \\ \sum_{i=1}^{t-j+1} F(i|\sigma_0=\delta_1)\gamma_{t-i,j-1|\sigma_0=\delta_1}, & \text{for } j=2,3,\cdots,t, \text{ and } t=2,3,\cdots,T, \end{cases}$$

and

$$\gamma_{t,j|\sigma_0=\delta_2} = \begin{cases} p_{22}^{t}, & \text{for } j=0 \text{ and } t=1,2,\cdots,T \\ p_{21}, & \text{for } j=1 \text{ and } t=1 \\ (t-1)p_{21}p_{12}p_{22}^{t-2} + p_{22}^{t-1}p_{21}, & \text{for } j=1 \text{ and } t=2,3,\cdots,T \\ \sum_{i=1}^{t-j+1} F(i|\sigma_0=\delta_2)\gamma_{t-i,j-1|\sigma_0=\delta_1}, & \text{for } j=2,3,\cdots,t, \text{ and } t=2,3,\cdots,T, \end{cases}$$

where

$$F(i|\sigma_0=\delta_1) = \begin{cases} p_{11}, & \text{for } i=1, \\ p_{12}p_{22}^{i-2}p_{21}, & \text{for } i=2,3,\cdots,t, \end{cases}$$

and

$$F(i|\sigma_0=\delta_2) = \begin{cases} p_{21}, & \text{for } i=1, \\ p_{22}^{i-1}p_{21}, & \text{for } i=2,3,\cdots,t. \end{cases}$$

Here, $F(i|\sigma_0=\delta_1)$ and $F(i|\sigma_0=\delta_2)$ are the probability that the first transition to state 1 occurs after i periods given that the initial regime is state 1 and 2, respectively.

Based on the regime switching option pricing theorem proposed in DPR (2002) and the payoff from the KKME contract displayed in Equation (4), the current value of $Loss_t^{KKME}$

that incorporates the business-cycle effect behind in housing prices can be valued by:

$$\begin{aligned}
\mathcal{L}^{RS}_{t|\sigma_0=\delta_k} &\equiv Put(K_{1,t}, t|\sigma_0 = \delta_k) - Put(K_{2,t}, t|\sigma_0 = \delta_k) \\
&= \sum_{j=0}^{t} \gamma_{t,j|\sigma_0=\delta_k}[K_{1,t}e^{-rt}N(-d_{2j}(K_{1,t})) - V_0 e^{-st} N(-d_{1j}(K_{1,t}))] \\
&\quad - \sum_{j=0}^{t} \gamma_{t,j|\sigma_0=\delta_k}[K_{2,t}e^{-rt}N(-d_{2j}(K_{2,t})) - V_0 e^{-st} N(-d_{1j}(K_{2,t}))] \\
&= \sum_{j=0}^{t} \gamma_{t,j|\sigma_0=\delta_k}\{[B_t + y]e^{-rt}[N(-d_{2j}(K_{1,t})) - (1 - L_R)N(-d_{2j}(K_{2,t}))] \\
&\qquad\qquad - V_0 e^{-st}[N(-d_{1j}(K_{1,t})) - N(-d_{1j}(K_{2,t}))]\},
\end{aligned} \tag{14}$$

where

$$d_{1j}(K_{i,t}) = \frac{\ln(V_0/K_{i,t}) + rt - st + 0.5\theta_j^2}{\theta_j},$$

$$d_{2j}(K_{i,t}) = d_{1j}(K_{i,t}) - \theta_j, \quad i = 1, 2.$$

Assume that the probabilities that the initial regime is state 1 and 2 are $P(\sigma_0 = \delta_1)$ and $P(\sigma_0 = \delta_2)$, respectively, the MI premium under the regime-switching framework, FP_0^{RS}, is given by the following expression:

$$FP_0^{RS} = \sum_{t=1}^{T} P_d(t)\{P(\sigma_0 = \delta_1)\mathcal{L}^{RS}_{t|\sigma_0=\delta_1} + P(\sigma_0 = \delta_2)\mathcal{L}^{RS}_{t|\sigma_0=\delta_2}\}. \tag{15}$$

We name this proposed pricing formula displayed in Equations (13)-(15) as the RS model.

4.2 Numerical analysis for the regime-switching property

Based on the KKME contract, this research further refines the valuation formula for MI contracts by incorporating the characteristics of business cycles in housing prices with the regime-switching option pricing model, since the housing prices are well observed to be cyclic. Here, we also report the fair premium as the Equivalent Annual Premium (EAP), which is calculated by Equation (9) with FP_0^{RS}. Table 2 exhibits the sensitivity of EAP to transition probabilities, i.e., p_{11} and p_{22}. The parameters in Table 2 are given by: $T = 30$, $L_V = 0.9$, $c = 5\%$, $\mathcal{R} = 0.5\%$, $L_R = 0.75$, $P_d(t) = 0.02$, $\mathcal{S} = 5\%$, $V_0 = 1,000,000$, $\delta_1 = 2\%$, and $\delta_2 = 6\%$. All parameters used in this table are identical to those in Table 1, except for the values of δ_1, δ_2, p_{11}, and p_{22}. These parameters are not required when pricing MI contracts under the assumption of the housing prices following the geometric Brownian motion as done in Table 1. We note that under this setting, state 1 indicates the low-volatility state, whereas state 2 represents the high-volatility state.

Given the initial regime is at state 1, the case of $p_{11} = 1$ indicates that the volatility always remains at the low variance state and thus the MI premium reduces to the price under the Black-Scholes model. The characteristic is clearly born out in Table 2. We find that all entries in the last row of Panel A are 85.10 bps, which not only is the lowest value observed from Panel A of Table 2, but also equals the KKME premium under the setting of $\sigma = 2\%$ and $P_d(t) = 0.02$ displayed in Table 1. Moreover, given the initial regime being state 1 and $p_{22} = 1$, the variance regime stays in regime 2 forever once it leaves regime 1. The time that the variance stays in the low-volatility regime shortens as the value of p_{11} declines. Therefore, for the case of $p_{22} = 1$ and the initial regime is at regime 1, the smaller the value of p_{11} is, the larger the RS premium will be. This is what we observed from the last column of Panel A in Table 2. As shown in Panel B of Table 2, these characteristics are shared within the cases of Panel B, in which the initial regime is at state 2, as well.

For any given value of p_{22} and initial regime, Table 2 also demonstrates that the RS fair premium declines as the value of p_{11} increases, because the probability of switching to the low-volatility state increases with p_{11}. Similarly, given the value of p_{11} and the initial regime, the probability of switching to the high-volatility regime increases as the value of p_{22} grows. This is the reason why the MI premium displays a non-decreasing function of

p_{22} when the value of p_{11} and the initial regime are given.

5 Empirical analysis

In this section, we apply the proposed approaches for MI contracts to empirically value the MI premiums in the U.S. market and compare the RS premiums with the GBM premiums. Our data, all-transactions house price index for U.S., come from the Federal Housing Finance Agency (FHFA). The sample period of quarterly house price index for both markets is from Quarter 1, 1975 to Quarter 4, 2010, so that there are 144 samples. To conduct empirical analysis, the returns of house price index are calculated as $(V_t - V_{t-1})/V_{t-1} \times 100\%$.

Table 3 reports the descriptive statistics of the returns in the U.S. market. The mean and standard deviation are 1.2232% and 1.2017%, and the skewness coefficient is -0.2015, which implies that the distributions of the housing returns skewed left in this market. Moreover, kurtosis coefficient is greater than 3, which shows that it is peaked distribution. The minimum and maximum are -2.5371% and 4.7331%, respectively.

We empirically estimate the parameters of the RS model and geometric Brownian motion (GBM) model for the U.S market by using maximum likelihood estimation. The parameters estimated are summarized in Table 4. The expected return of the house price index estimated from the two-state RS model, i.e., μ_1 and μ_2, are 1.1213% and 1.3985% for each state. The volatility estimated for each state, i.e., δ_1 and δ_2, are 0.6416% and 1.7894%. On the other hand, the expected return and volatility estimated from the GBM model are 1.232% and 1.2064%. Both the two parameters, i.e., μ and σ, estimated from the GBM model are observed to lie in between its corresponding state-dependent parameters, i.e., μ_1, μ_2, δ_1, and δ_2, from the RS model. Based on the estimated parameters, we find that the low-volatility state is with a high expected return, whereas the high-volatility state is with a low expected return. It indicates that the housing prices are less volatile in depression but more volatile in prosperity, which is consistent with the empirically findings in the literature.

Figures 3 compares the historical returns and the estimated probability of being in state 2. Clearly, the figure shows that the inferred probability of state 2 (high variance state) is close to 1 when the housing market is volatile. On the contrary, the probability of state 2 approaches to 0 when the market is relatively stable. Figures 3 indicates that the dynamics of housing prices exist the business cycle, which means that the ability of the regime-switching model in identifying the different level of volatilities is well-defined. Hence, the RS model is able to capture the change in the state. Accordingly, the RS model is more appropriate than the GBM model for modeling the house prices when valuing the MI contracts.

Table 5 displays the equivalent annual premium (EAP) for the U.S. market based on the data ranged from Quarter 1, 1975 to Quarter 4, 2010. Specifically, we use the 1-year Treasury rate as the risk-free interest rate $\mathcal{R}$ reported in the Datastream database, which is 0.29% on December 31, 2010. Based on the Datastream database, the mortgage rate c is set to be 4.61% for U.S. market, which is the fixed 30-year contract interest rate on December, 2010. According to the information of Global Property Guide, the housing yield $\mathcal{S}$ is 5% in the U.S. market. Based on the results in Table 4, the volatility σ used to calculate the GBM premium is 1.2064%. The volatilities for the two states used to compute the RS premium are $\delta_1 = 0.6416\%$ and $\delta_2 = 1.7894\%$. Moreover, based on the results displayed in Table 4, the transition probability p_{11} and p_{22} are given by 0.9823 and 0.9856 in the U.S. market. The probabilities of state 1 and 2, i.e., $P(S_0 = 1)$ and $P(S_0 = 2)$, are given by 0.0046 and 0.9954, respectively. Finally, the unconditional default probability $P_d(t)$ is set to 0.02 on average according to the information of Mortgage Bankers Association (MBA), which is reported for the mortgage delinquency rates on Q4

2010. The other parameters without mention are given by: T=30, V_0=1,000,000, and L_R=0.78.

According to the Mortgagee Letter reported by Department of Housing and Urban Development on February 14, 2011, it is useful to compare the RS premium with the market data. Given that upfront mortgage insurance premium is 100 bps, the loans term is more than 15 years, and the loan-to-value is less than or equal to 95%. The annual premium is 110 bps, which is increased by the 25 bps to the annual mortgage insurance premiums. It is close to 122.46 bps, which is calculated by the RS model in the case of L_V=0.95 and $P_d(t)$=0.02 in Table 5. We also observe that the EAP calculated by the RS model are higher than that from the GBM model. It implies that during the subprime crisis the insurer should charge more premiums based on the RS model to reduce the potential loss, which results from the undervalued fair premiums without considering the volatility change. Again, it indicates that valuing MI contracts without incorporating the business-cycle property of house prices may underestimate the fair premium.

Table 5 also reports the sensitivity of the EAP to the unconditional default probability $P_d(t)$ and loan-to-value ratio L_V. As expected, the EAP calculated by both of models grows with the increase in the loan-to-value ratio L_V and unconditional default probability $P_d(t)$.

6 Conclusions

This paper first refines the pricing formula of MI contracts proposed in BKU (2006) by re-identifying the setting of conditional losses to insurers in the case of default at time t, namely the KKME contract. Hence, we propose a refined pricing formula of MI contracts, GBM model. The numerical results show that the re-identification affects significantly the potential losses to insurers. It causes that the fair premiums calculated by the KKME contract are larger than those calculated by the BKU contract. We note that the pricing deviation may be one of reasons that result in the losses of the insurance companies. Furthermore, business cycle is a property that can be observed from housing market usually. It is appropriate to assume that the housing price process follows the Markov regime-switching model for capturing the business-cycle feature behind the housing market.

We incorporate the Markon regime-switching option pricing model proposed in DPR (2002) into the KKME contract and provide a closed-form formula for pricing MI contracts, namely RS model. Numerical results show that the parameters of RS model, i.e., volatilities under different states and the transition probabilities, influence the premiums significantly. Additionally, results of real-time analysis show that the ability of the RS model in identifying the business cycle underlying home prices is superior for the U.S. market, and the fair premium of MI contracts is higher than that of the GBM model in Quarter 4, 2010 when we take the business-cycles property behind housing prices into account.

In recent years, many mortgage insurance companies have been reported to suffer from great losses. Our results imply that incorporating the regime-switching property in valuing MI contracts may facilitate to reduce the losses of insurance companies which are commonly observed in the last three years.

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Table 1. Equivalent annual premiums (EAP) calculated from the BKU contract and KKME contract

(basis points)

		$P_d(t)=0.005$		Pricing	$P_d(t)=0.01$		Pricing	$P_d(t)=0.02$		Pricing	$P_d(t)=0.04$		Pricing
		BKU	KKME	deviation	BKU	KKME	deviation	BKU	KKME	deviation	BKU	KKME	deviation
T	20	1.63	2.03	-19.70%	3.27	4.07	-19.66%	6.53	8.13	-19.68%	13.07	16.26	-19.62%
	25	8.07	9.18	-12.09%	16.15	18.35	-11.99%	32.29	36.70	-12.02%	64.58	73.40	-12.02%
	30	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
Σ	2%	19.28	21.28	-9.40%	38.55	42.55	-9.40%	77.10	85.10	-9.40%	154.20	170.20	-9.40%
	4%	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
	6%	25.42	27.17	-6.44%	50.84	54.33	-6.42%	101.68	108.66	-6.42%	203.35	217.33	-6.43%
L_V	0.85	15.35	16.83	-8.79%	30.70	33.66	-8.79%	61.40	67.32	-8.79%	122.79	134.64	-8.80%
	0.90	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
	0.95	28.89	31.07	-7.02%	57.77	62.13	-7.02%	115.55	124.27	-7.02%	231.09	248.54	-7.02%
c	4%	14.94	16.09	-7.15%	29.87	32.17	-7.15%	59.74	64.35	-7.16%	119.49	128.70	-7.16%
	5%	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
	6%	29.92	32.64	-8.33%	59.84	65.28	-8.33%	119.68	130.56	-8.33%	239.36	261.13	-8.34%
$\mathcal{R}$	0.25%	26.19	28.21	-7.16%	52.39	56.42	-7.14%	104.77	112.84	-7.15%	209.55	224.68	-6.73%
	0.5%	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
	0.75%	17.72	19.39	-8.61%	35.45	38.77	-8.56%	70.89	77.54	-8.58%	141.79	155.09	-8.58%
L_R	0.25	20.16	21.75	-7.31%	40.32	43.51	-7.33%	80.64	87.01	-7.32%	161.28	174.02	-7.32%
	0.75	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%
	0.90	21.71	23.56	-7.85%	43.43	47.11	-7.81%	86.85	94.23	-7.83%	173.71	188.45	-7.82%

Note: (1) Parameters which are not directly specified in each case are given by: T = 30, $V_0 = 1{,}000{,}000$, $L_V = 0.9$, c = 5%, $\mathcal{R}$ = 0.5%, $\mathcal{S}$ = 5%, $\Sigma = 4\%$, and $L_R = 0.75$.

(2) The fair premium of the KKME model is calculated by Equations (6)-(8), with the settings of $K_{1,t} = B_t + y$ and $K_{2,t} = (1-L_R)(B_t + y)$.

(3) The fair premium of the BKU model is calculated by Equations (6)-(8), with the settings of $K_{1,t}^{BKU} = B_{t-1}$ and $K_{2,t}^{BKU} = (1-L_R)B_{t-1}$.

(4) We assume the fair premium to be paid annually and transfer it to the equivalent annual premium (EAP) by Equation (9).

(5) EAP is displayed in basis points.

Table 2. Sensitivity of equivalent annual premiums (EAP) calculated from the RS model to transition probabilities

(**basis points**)

Panel A: Given that the initial regime is state 1.

		p_{22}				
		0.0	0.25	0.5	0.75	1.0
p_{11}	0.0	97.49	99.19	101.41	104.40	108.66
	0.25	95.69	97.41	99.76	103.14	108.43
	0.5	93.27	94.89	97.26	101.04	107.97
	0.75	89.88	91.08	93.06	96.82	106.61
	1.0	85.10	85.10	85.10	85.10	85.10

Panel B: Given that the initial regime is state 2.

		p_{22}				
		0.0	0.25	0.5	0.75	1.0
p_{11}	0.0	97.09	98.86	101.15	104.25	108.66
	0.25	95.35	97.15	99.61	103.14	108.66
	0.5	93.00	94.73	97.26	101.29	108.66
	0.75	89.71	91.08	93.33	97.60	108.66
	1.0	85.10	85.38	85.92	87.39	108.66

Note: (1) Parameters which are not directly specified in each case are given by: T = 30, $V_0 = 1{,}000{,}000$, L_V = 0.9, c = 5%, $\mathcal{R}$ = 0.5%, $\mathcal{S}$ = 5%, L_R = 0.75, and $P_d(t) = 0.02$. Moreover, the volatilities for the two states are given by: $\delta_1 = 2\%$ and $\delta_2 = 6\%$.

(2) The fair premium of the RS model is calculated by Equations (13)-(15). We assume this fair premium to be paid annually and transfer it to the equivalent annual premium (EAP) by Equation (9).

(3) EAP is displayed in basis points.

Table 3. Descriptive statistics of returns in the U.S. market

(%)

	Mean	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
U.S.	1.2232	1.2107	-0.2015	4.4913	-2.5371	4.7331

Note: Our data, U.S. house price index, is obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010, so that there are 144 observations. Statistics for the U.S. market are calculated from returns of the corresponding house price index.

Table 4. Parameter estimates of the RS model and GBM model

	μ_1	μ_2	p_{11}	p_{22}	σ_1	σ_2	μ	σ
RS	1.1213	1.3985	0.9823	0.9856	0.6416	1.7894		
	(0.0715)	(0.2502)	(0.0143)	(0.0172)	(0.0526)	(0.1850)		
GBM							1.2232	1.2064
							(0.1009)	(0.0713)

Note: Our data, U.S. house price index, is obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010. The RS and GBM model indicate Markov regime-switching model and geometric Brownian motion model, respectively. Figures in parentheses are standard deviations.

Table 5. Equivalent annual premiums (EAP) estimated from the U.S. market

(**basis points**)

		$P_d(t)=0.005$		$P_d(t)=0.01$		$P_d(t)=0.02$		$P_d(t)=0.04$	
		GBM	RS	GBM	RS	GBM	RS	GBM	RS
L_V	0.85	14.90	15.99	29.80	31.98	59.60	63.96	119.20	127.93
	0.90	22.17	22.97	44.35	45.94	88.70	91.88	177.39	183.76
	0.95	30.11	30.62	60.22	61.23	120.45	122.46	240.90	244.93

Note: (1) EAP is calculated based on estimates and data from the U.S. market. Specifically, we use the 1-year Treasury rate as the risk-free interest rate $\mathcal{R}$, which is 0.29%. Based on the information of the Datastream database, the mortgage rate c are set to be 4.61%. Accordingly to the information of Global Property Guide, the housing yield S is 5% in the U.S. market. Other parameters are given by: $T = 30$, V_0 =1,000,000, and $L_R = 0.78$.

(2) The fair premium of the GBM model is calculated by Equations (6)-(8). Based on the results in Table 4, the volatility σ estimated from the GBM model and is 1.2064% for the U.S. market.

(3) The fair premium of the RS model is calculated by Equations (13)-(15). Based on the results of Table 4, the volatilities for the two states are $\delta_1 = 0.6416\%$ and $\delta_2 = 1.7894\%$ in the U.S. market.

(4) We assume the fair premium to be paid annually and transfer it to the equivalent annual premium (EAP) by Equation (9). EAP are displayed in basis points.

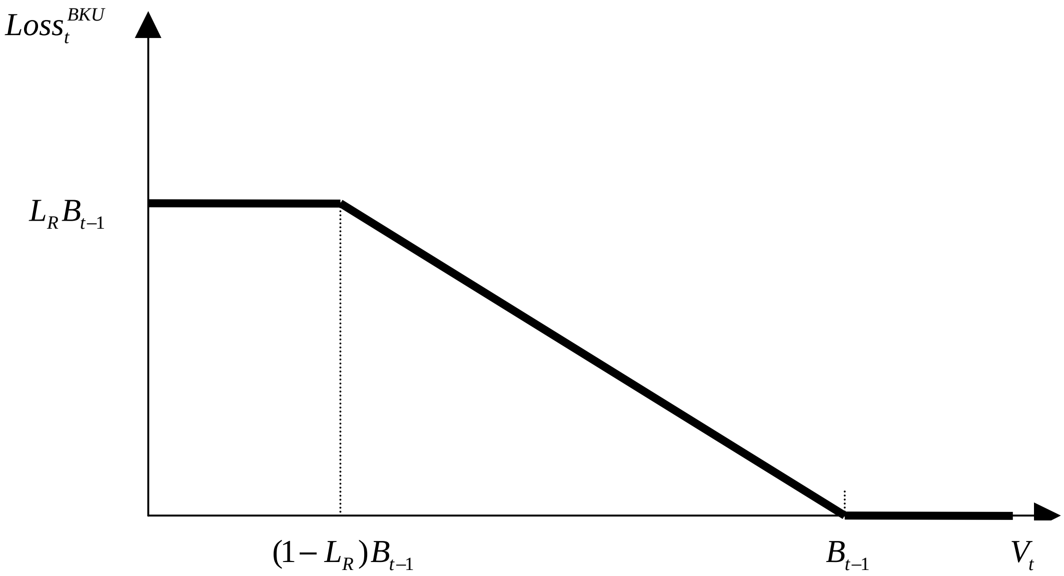

Figure 1. The payoff in BKU approach when the borrower defaults at time *t*

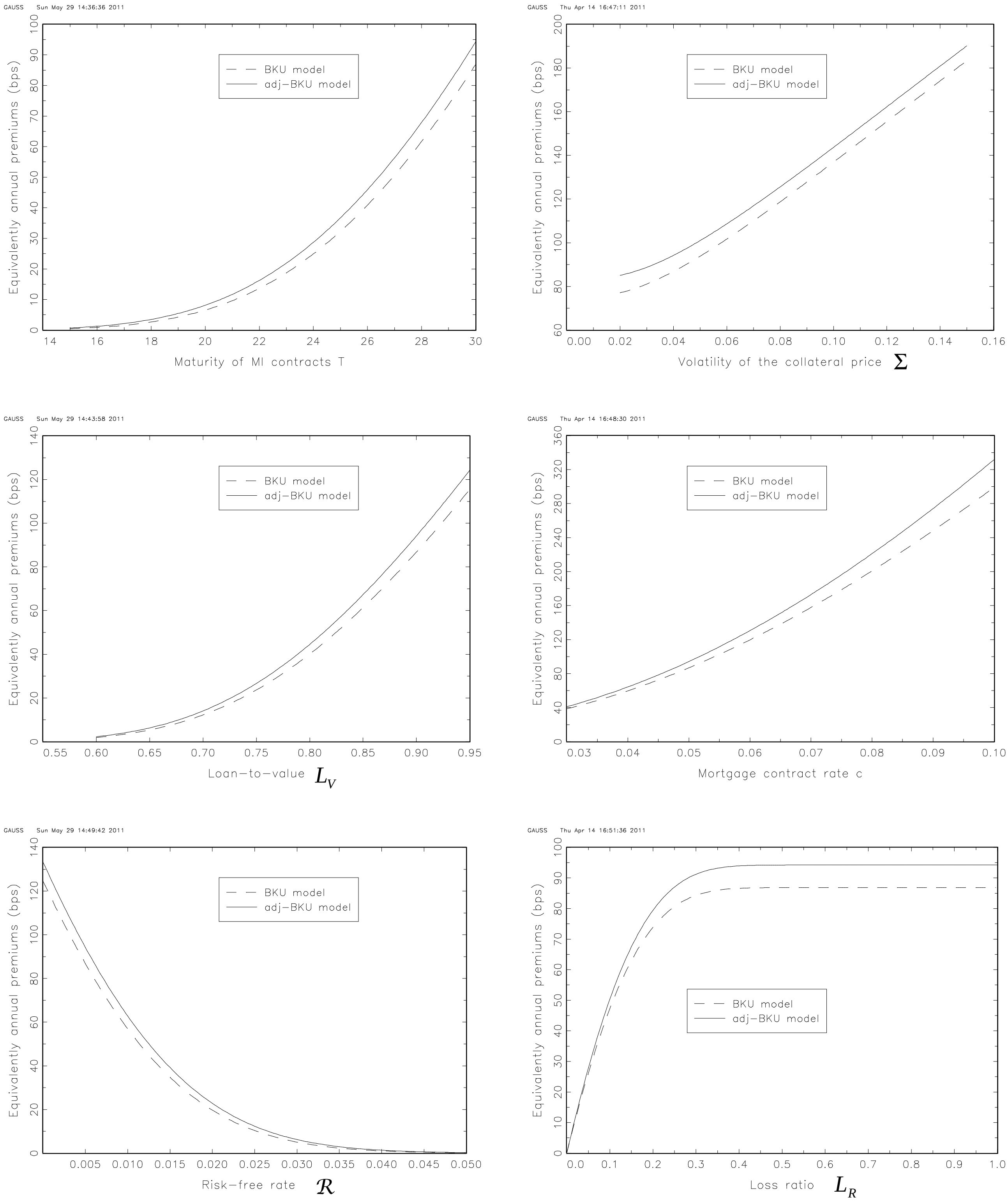

Figure 2. The impacts of various parameters on equivalent annual premiums (EAP)

The impacts of various parameters, including the maturity *T*, volatility of the collateral price Σ, loan-to-value L_V, mortgage rate *c*, risk-free rate $\mathcal{R}$, and loss ratio L_R, on equivalently annual premiums (EAP). Parameters which are not directly specified in each figure are given by: $T = 30$, V_0 = 1,000,000, L_V = 0.9, c = 5%, $\mathcal{R}$ = 0.5%, S = 5%, Σ = 4%, L_R = 0.75, and $P_d(t) = 0.02$.

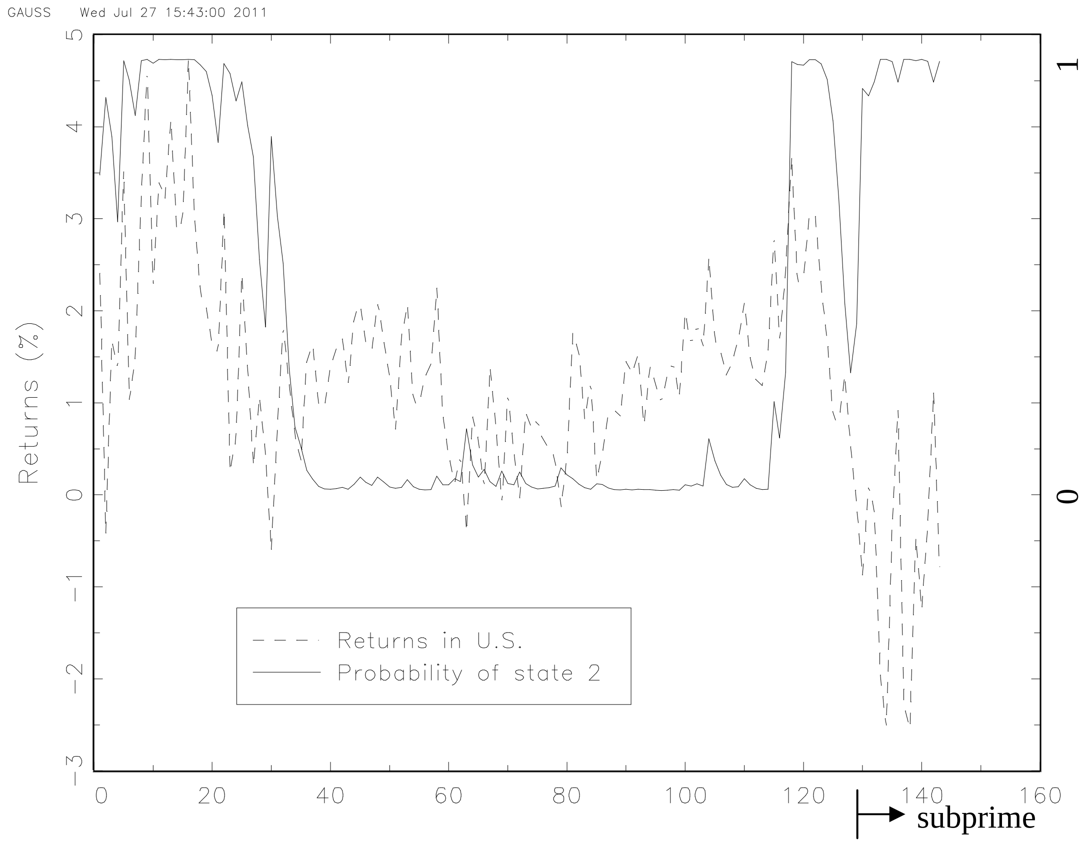

Figure 3. Inferred regime probability of being in state 2 (high variance state) estimated from the RS model for the U.S. market.

Returns in the U.S. housing market are calculated by the quarterly U.S. house price index, which are obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010. This figure shows that the inferred probability of state 2 (high variance state) is close to 1 when the U.S. housing market is volatile. In the contrary, the probability of being in state 2 approaches to 0 when the market is relatively stable. It is also observed that the RS model clearly identifies the period of subprime crisis as the high variance state in the U.S. market. It indicates that the superior ability of the RS model in identifying the business cycle.