東海大學管理學院財務金融研究所

碩士論文

壞 **Beta,** 好 **Beta** 與違約風險異象

Bad Beta, Good Beta, and Default Risk Anomaly

(初稿)

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中華民國 100 年 7 月

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中文摘要

過去研究指出,高違約風險的公司有異常低的股票報酬。本研究使用 Campbell and Vuolteenaho (2004)所提出的兩 Beta 模型去解釋此異常現象。我們將資本資產定價模型 Beta 拆解成現金流量 Beta 與折現率 Beta。Campbell and Vuolteenaho (2004)指出前者的風 險價格高於後者。我們使用 Duffie, Saita, and Wang (2007)去衡量每一間公司的違約風險。 另外,為了探討違約風險,我們控制了公司的融資限制風險;我們使用主成分分析建立 一個新的融資限制指數。實證上,我們發現當公司違約風險增加時,會有較小的現金流 量 Beta 與較高的折現率 Beta。我們發現現金流量 Beta 的風險溢酬為正,折現率 Beta 的 風險溢酬為負。另外,我們想知道違約風險是否會影響現金流量 Beta 與折現率 Beta, 我們發現違約機率會降低現金流量 Beta 與增加折現率 Beta。

關鍵字:違約風險、破產風險、Beta、融資限制

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Abstract

Recent studies suggest that stocks with higher default risk earn anomalously low returns. This paper explains the "default risk anomaly" using Campbell and Vuolteenaho's (2004) two beta model. We break the CAPM beta into two components: cash-flow beta and discount-rate beta. Campbell and Vuolteenaho (2004) suggest that the former has the higher price of risk than that of the latter. We measure the default probability of firms using the model by Duffie, Saita, and Wang (2007). To properly control default risk, we control the financial constraint risk. We create a new financial constraint index by principal component approach. Empirically, we find that stocks with higher default risk have small cash-flow beta and large discount-rate bet. The risk premium of cash-flow beta is positive and that of discount-rate beta is negative but no statistically significant. We test whether default risk affects cash-flow and discount-rate beta. We find that cash-flow beta decreases and discount-rate beta increases as default risk rises.

Key words: Default; Bankruptcy; Beta; Financial Constraint

Contents

Table Contents

Figure contents

Figure 1. Risk Premium of Two Betas 29

I. Introduction

It has been well acknowledged in the finance literature that stocks with higher risks should have higher returns. In the early 1960s, Sharpe (1964) and Lintner (1965) introduced the Capital Asset Pricing Model (CAPM) to compute the expected return of equity. Recent studies provide the evidence that firms with higher default risk have the lower returns; the traditional CAPM does not explain this anomaly. For example, Dichev (1998) used Z-score (Altman (1968)) and O-score (Ohlson (1980)) to measure the default risk of each firm, and found that firms with high bankruptcy risks earn lower returns. Campbell, Hilscher, and Szilagyi. (2008) adopted the dynamic logit model devised by Shumway (2001) to estimate the probability of bankruptcy or failure. Their empirical results continue to suggest that financially-distressed stocks are underperformed and earn lower returns. Alternatively, Garlappi and Yan (2011) used Moody's KMV's default probability to examine this anomaly between the default risk and average returns.

In this article, we use Campbell and Vuolteenaho's (2004) model to explain this anomaly. Campbell and Vuolteenaho (2004) decompose the traditional beta into two components: cash-flow beta and discount-rate beta. They claim the discount-rate beta as "good beta" because of the poor stock returns driven by discount rates are compensated by future returns. The cash-flow beta captures the cash flow news about the future investment opportunities of companies, so they claim the risk premium of cash-flow beta is higher than that of discount-rate beta. Therefore, they found that the value stock outperforms the growth stock because of the risk premium of cash-flow beta is positively higher than discount-rate beta. This can explain the value premium puzzle that value stocks with higher cash-flow beta causes the higher returns. We argue that Campbell and Vuolteenaho's (2004) two beta model can be applied to explain the default risk anomaly.

Interestingly, we find that high (low) financially distressed firms have high discount-rate

(cash-flow) beta. The risk premium of cash-flow beta is positively and that of discount-rate beta is negative but not statistically significant. We also report the result whether default risk or financial constraint risk affect cash-flow beta and discount-rate beta after we control some variables from Lewellen and Nagel (2006). Following Fama and MacBeth (1973), we find that default risk is one of several factors that cause cash-flow betas to decrease and discount-rate betas increase as default risk rises. It provides some evidence that firms with higher default risk have lower cash-flow beta which causes lower equity returns.

In order to evaluate the level of firm's default risk, we use the approach of Duffie, Saita, and Wang (2007). Duffie et al. (2007) use the Poisson intensity model to characterize the default probability. In addition, Shumway (2001) considers market variables (covariates) into the default probability model, and finds that the rate of accuracy of the default probability model with market variables is higher than the model without the accounting variables. In addition, we add some accounting covariates as devised by Duan, Sun, and Wang (2011a). We empirically find that the covariates are significant and consistent with Duffie et al. (2007) and Duan et al. (2011a), and Pseudo- R^2 of our parsimonious model is higher than the model of Duffie et al. (2007).

 To properly control the financial distress risk, we consider the issue of financial constraint risk. Firms with insufficient cash flow that attempt to deal with the operating expense may cause default risk rise. It is possible that firms with the higher default risk leads to no loanable funds lent by banks. Sepcifically, we create a financial constraint index composed of KZ (Lamont, Polk, and Saá – Requejo (2001)), WW (Whited and Wu (2006)), and SA index (Hadlock and Pierce (2010)) by using the principal component approach. We find that the financial constraint index summarized by the first principal component explains about 70% to 80% of the sample variance; it preserves the majority of information from the three financial constraint indices.

 We create a 25 financial constraint index- and default probability-sorted portfolios after estimating the default probability and financial constraint level of each firm. Then we calculate cash-flow beta and discount-rate beta and average return of each portfolio. We find that the cash-flow betas and discount-rate betas display a humped shape, and the average returns show the same pattern with cash-flow and discount-rate beta after we control the financial constraint risk. It is consistent with Garlappi and Yan (2011), who suggest that there is a hump-shaped relationship between equity beta and default risk. We show that average returns decrease and the CAPM betas increase as default risk rises, and the increase in the CAPM beta can be mainly attributed to the increase in discount-rate beta. Since the CAPM beta can be decomposed into cash-flow beta and discount-rate beta that contribute positive and zero risk premiums, respectively, large discount-rate beta in high financially-distressed firms seems to explain the anomaly why stocks with higher default risk can expect the lower returns.

 The framework of the paper is as follows. In section 2, we report the context of recent studies about default risk and financial constraint risk. In section 3, we describe the methodology about estimating cash-flow beta and discount-rate beta, default probability, and a new financial constraint index. In section 4, we show our data source and some explanation of covariates. In section 5, we report our empirical results. In section 6, we report our conclusions.

II. Related literature

A. Review of Default Risk Prediction

Credit risk has been a hot issue in recent years. The model of credit risk is categorized into two groups: reduce-form model and structural model. Primary research of bankruptcy can be traced back to the 1970s, when Beaver (1966) used leverage ratio to predict the failure of the firms. Altman (1968) adopted a statistic technique called multiple discriminant analysis (MDA) to create a discriminant function that is composed of firm characteristics. He chose thirty-three companies in each group from bankrupted and survival firms as the sample. The sample consists of manufacturing firms from Moody's industrial Manuals and annual reports during 1946-1965. The five financial variables are selected from twenty-two financial ratios which are adopted in other researches. The Z score, calculated via the discriminant function, can help us know the bankrupt probability of each firm. He suggests that a Z score lower than 1.81 is classified in the bankruptcy group, and one higher than 2.99 is classified in the non-bankruptcy group. The score between 1.81 and 2.99 is the gray area that cannot be classified correctly. But the Z score has no ability to predict for the long period prior to bankruptcy. Therefore Altman, Haldeman, and Narayanan (1977) show a new discriminant model (Zeta model) that improves the shortcomings of the Z-score model. The Zeta model solves the problem of prediction ability and can also analyze the retailing firms.

 Even Z-score and Zeta models have a prominent prediction power with regard to bankrupted firms; there are some problems in using the long- term MDA approach. For example, the variance-covariance matrices of covariates have to be the same and normally distributed between the bankruptcy and non-bankruptcy group. The discriminant function is invalid if violating the requirement of using the MDA. Hence, Ohlson (1980) uses the logit model as the estimating tool, avoiding the limits of the MDA. The sample is extended from the manufacturing industry to others- except for financial, transportation and utilities. He chooses the covariates whose financial ratios are used frequently in the literature. The coefficients of covariates are estimated by the Maximum likelihood estimation. The score calculated from the logit model represents the probability of bankruptcy. Ohlson then chooses an optimal cutoff point that minimizes the prediction error. Even the accurate ratio of the O-score model is higher than for other prediction models, and there exist some problems that are not taken into account in the model.

 Shumway (2001) develops a discrete-time hazard model that solves the problems that occurred in the logit model. The hazard model considers the risk of each firm at every time period and incorporates the time-varying variables. The prediction performance of the hazard model combined accounting and market variables, and is more accurate than the model with market variables only. For greater robustness, he re-estimates the other forecasting models by using the different variables and compares the forecasting accuracy of each model with the hazard model. Thus, he indicates that the hazard model is a more appropriate way to forecast bankruptcy. However, the covariates of the hazard model are both market and accounting variables; it seems unreasonable to use past information to forecast the future status of firms. Duffie, Saita, and Wang (2007) provide the double stochastic intensity model that incorporates the time-varying covariates of firm-specific and market. They classify the firms into five groups based on exit type: bankruptcy, default, failure, merger-acquisition, and other. The accuracy ratio of out-sample prediction compared with other models is higher at approximately 88%. Another advantage of the intensity model is that the hazard rate can be predicted from the future periods to the date of the firms that exit.

B. Financial Constraint Index

In recent years, many methods have been developed to show how firms are financially constrained or not (please check this correction- again I am not sure of the original meaning). For example, the following measures are used frequently, including payout ratio, firm size, credit rating, etc. Fazzari, Hubbard, and Petersen (1988), (hereafter FHP (1988)), use payout ratio to classify the firms which are divided into three groups. FHP (1988) indicate that firms that are less financially constrained have a higher payout ratio, and test the effect of sensitivity of investment to cash flow between the firms which are financially constrained and those which are not. They chose manufacturing firms as their sample- from 1970 to1984. Firms with less than 10 percent payout ratio for at least ten years were categorized into Group 1. Firms with a payout ratio between 10 percent and 20 percent are categorized into Group 2, and other firms are categorized into Group 3. They find that more constrained firms have higher sensitivity of investment to cash flow than those less constrained.

However, Kaplan and Zingales (1997) do not support the result that FHP uses the investment to cash flow sensitivity as a criterion to measure financially constraints. In order to prove their point, they chose sample Group 1- these firms have a lower payout ratio from FHP (1988). Kaplan and Zingales (1997) find that the less constrained firms have higher sensitivity of investment to cash flow than the more constrained ones. As a result, they conclude that measuring financial constraints by using sensitivity of investment to cash flow is inappropriate.

 So far we have found many firm characteristics are used to sort by various researchers. We do not actually observe any approaches which can be used to compare the level of financial constraint in each firm yet. Until Lamont et al. (2001) constructed a financial constraint index called "KZ index", who estimated the sample from Kaplan and Zingales (1997) by using order logit model. The KZ index is composed of five estimated coefficients, which are leverage, cash flow, cash, dividends, and Tobin's Q. All firms in their sample are divided into three groups based on firm size and KZ index. A firm size in the top 33% of all firms is regarded as the big firm. There are totally nine groups: Low KZ/ small (LS), Low KZ/medium (LM), Low KZ/big (LB), medium KZ/small (MS), medium KZ/medium (MM), medium KZ/ big (MB), High KZ/small (HS), High KZ/medium (HM), and High KZ/ big (HB). They regress returns of nine groups on returns of three proxy portfolios which are proxies of the market factor, size factor, and constraints factor. They find that there exists a constraint factor in stock returns. In other words, the stock returns of constrained firms moved

together with the stock returns of other constrained firms.

 Even where there is a benchmark of distinguishing the constrained firms, there exist some problems in the KZ index. To solve these problems, Whited and Wu (2006) developed another financial constraint index that can sort the firms more accurately. They estimate an investment Euler equation by using a generalized method of moments (GMM). Then they create an index which consists of six factor, including cash flow, long-term debt, size, sales growth, industry sales growth, and dividend dummy. The advantage of the Whited and Wu index (WW index) is that it solves the sample selection problem and measurement error of Tobin's Q. After using the new index that they create to sort the sample, they find that constrained firms are smaller, underinvest, have lower cash flow, analyst coverage, and have rare bond ratings relative to the unconstrained firms. Contrary to the new index, the constrained firms are bigger, have more analyst coverage and bond ratings.

 Hadlock and Pierce (2010) construct a new index which consists of firm age and size. They follow the methodology of Kaplan and Zingales (1997) by using the qualitative classification to select the constrained firms. After firms are chosen from the annual report and financial filing of firms, they categorize the samples into five groups. In order to estimate the KZ index, they follow the approach of Lamont et al. (2001). The signs of estimated coefficients that included cash flow, Q, and debt are consistent with the original KZ index, but the signs of dividends and cash differ. They also consider many situations of the sample; however, some estimated coefficients are still with different signs relative to the original KZ index. After the examination of stability of the coefficient, they doubt the validity of the KZ index. Another index of financial constraint they estimate is advocated by Whited and Wu (2006). They estimate the WW index by using the order logit model, finding that some coefficients are still significant and have the same signs (cash flow and leverage). But some estimated coefficients are insignificant (firm sales growth and industry sales growth) or opposite signs (dividends dummy). After the test they do above, they find that the two variables "firm size" and "age" are incapable of predicting the status of constrained firms. They also estimate the coefficients of firm size and age of the sample by order logit model. The signs of estimated coefficients are all the same and significant, even considering other conditions. Therefore, Hadlock and Pierce (2010) suggest that the new index is a simple and reasonable approach of measuring the constrained firms.

III. Methodology

A. VAR System

We follow Campbell and Vuolteenaho (2004) to decompose the return into cash-flow and discount-rate news. They use a loglinear approximate present-value relation that allows for time-varying discount rates developed by Campbell and Shiller (1988a). They define the approximation of log return on the dividend-paying asset at time $t + 1$ as the log equity price and dividends at time $t + 1$ and subtract the log equity price at time t. P and D denotes the equity price and dividends respectively; the approximation equation is $r_{t+1} \equiv$ $\log(P_{t+1} + D_{t+1}) - \log(P_t)$. Then they expand the mean log dividend-price ratio, $(\overline{d_t - p_t})$, by a first-order Taylor expansion. The result of the Taylor expansion approximation of log return on the dividend-paying asset is $r_{t+1} \approx k + \rho p_{t+1} + (1 - \rho)d_{t+1} - p_t$. The parameters of ρ and k are defined by $\rho = 1/(1 + \exp(\overline{d_t - p_t}))$ and ρ)log (1/ ρ – 1). As the dividend-price ratio is constant, ρ equal the ratio of the ex-dividend to the cum-dividend equity price, that is $\rho = P/(P + D)$. They solve the Taylor expansion approximation iteratively, and consider the "no-infinite-bubbles" terminal condition that $\lim_{i\to\infty}\rho^j(d_{t+i}-p_{t+i})=0$. Then they take the approximation expectations, and subtract the dividend; they have the equation:

$$
p_t - d_t = \frac{k}{1 - \rho} + \mathcal{E}_t \sum_{j=0}^{\infty} \rho^j \left[\Delta d_{t+1+j} - r_{t+1+j} \right]
$$
 (1)

Where Δd is the log dividend growth. Campbell (1991) substitute the equation (1) into the approximate return equation, and they get

$$
r_{t+1} - \mathbf{E}_t r_{t+1} = (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}
$$
(2)
= $N_{CF,t+1} - N_{DR,t+1}$

Where N_{CF} represent the news about future cash flows, which includes the news of dividend paying or the future consumption of investor; N_{DR} represent the news about future discount rates.

 Campbell et al. (2004) estimate the equation (2) by using a vector autoregressive (VAR) model. They estimate the terms $E_t r_{t+1}$ and $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}$ first, then they can get the cash-flow news more easily. The first-order VAR model is

$$
z_{t+1} = \alpha + \Gamma z_t + u_{t+1} \tag{3}
$$

Where z_{t+1} is a k × 1 vector, and r_{t+1} is the first element of z_{t+1} . α is the k × 1 vector of constant term, Γ is a k \times k matrix of parameters, and u_{t+1} is an i.i.d. $k \times 1$ of shocks. We can calculate the cash-flow and discount-rate news after we get the estimates of vector, Γ , and shock vector, u_{t+1} . The following are the functions of news

$$
N_{CF,t+1} = (\mathbf{e1}' + \mathbf{e1}'\lambda) u_{t+1}
$$

$$
N_{DR,t+1} = \mathbf{e1}'\lambda u_{t+1}
$$

Where λ captures the VAR shocks that we can use it to calculate the cash-flow and discount-rate news, and it is defined as $\lambda = \rho \Gamma (I - \rho \Gamma)^{-1}$. We estimate the cash-flow and discount-rate betas using the following beta estimators.

$$
\hat{\beta}_{i,CF} = \frac{\widehat{\text{Cov}}(r_{i,t}, \widehat{N}_{CF,t})}{\widehat{\text{Var}}(\widehat{N}_{CF,t} - \widehat{N}_{DR,t})}
$$

$$
\hat{\beta}_{i,DR} = \frac{\widehat{\text{Cov}}(r_{i,t}, \widehat{N}_{DR,t})}{\widehat{\text{Var}}(\widehat{N}_{CF,t} - \widehat{N}_{DR,t})}
$$

Campbell et al. (2004) estimate two betas using the data over the entire period from 1929 to 2001. We consider the time-varying effect of two betas, estimated from moving a 36-month window for stocks at the end of each month.

B. Default Intensity Model

We use the default intensity model introduced by Duffie et al. (2007) to compute the default probability. The default intensity λ is an independent Possion process where the first event time τ of firm is merged, delisted, or bankrupted. λ_{it} is the conditional mean arrival rate of default of firm *i* at time *t*. U_{it} is the firm-specific vector of covariates for firm *i* from the time of data appear at t_i to the firm exit at T_i , and V_t is the vector of macroeconomic covariates. We suppose that $\lambda_{it} = \Lambda(W_{it}, \theta)$, where $W_{it} = (1, U_{it}, V_t)$ is a vector of the all covariates.

The default intensity of firm *i* at time *t* form a proportional hazards function

$$
\Lambda(w,\theta)=e^{\beta_1w_1+\cdots+\beta_nw_n}
$$

 θ is the parameter vector of β . We create a default indicator (D_{it}) equal one when firm exit, otherwise is zero. Follow Proposition 2 of Duffie et al. (2007), the maximum likelihood estimator (MLE) of parameters (γ, θ) form

$$
\mathcal{L}(\gamma, \theta | W_{it}, D_{it})
$$

= $\mathcal{L}(\gamma | W_{it}) \mathcal{L}(\theta | W_{it}, D_{it})$
= $\mathcal{L}(\gamma | W_{it}) \prod_{i=1}^{m} (e^{-\sum_{t=t_i}^{T_i} \lambda_{it} \Delta t} \prod_{t=t_i}^{T_i} [D_{it} \lambda_{it} \Delta t + (1 - D_{it})])$

We also estimate the vector γ of parameters, which is considered the time-series effect of U_{it} and V_t .

C. Financial Constraint Index

In order to measure the level of the financial constraint of each firm, we consider three famous financial constraint index, these are KZ, WW and SA index. Baker and Wurgler (2006) use the principal component analysis to mix several investor sentiment variables into an investor sentiment index. So we follow Baker and Wurgler (2006) to create a financial constraint index which is composed of the three indices. The following formulas are KZ, WW and SA index

$$
KZ = -1.001909 \cdot CF_{it} + 3.139193 \cdot TLTD_{it} - 39.36780 \cdot TDIV_{it} - 1.314759 \cdot CASH_{it}
$$

$$
+ 0.2826389 \cdot Q_{it}
$$

where CF_{it} is the ratio of cash flow to property, plant and equipment (PPE). $TLTD_{it}$ is the ratio of debt to total capital. $TDIV_{it}$ is the ratio of dividends to PPE. $CASH_{it}$ is the ratio of cash to PPE. Q_{it} is Tobin's q.

$$
WW = -0.0091 \cdot CF_{it} - 0.062 \cdot DIVPOS_{it} + 0.021 \cdot TLTD_{it} - 0.044 \cdot LNTA_{it}
$$

$$
+0.102 \cdot ISG_{it} - 0.035 \cdot SG_{it}
$$

where $DIVPOS_{it}$ is the dummy that equals the value one if firms pay cash dividends. $LNTA_{it}$ is the natural log of total assets. ISG_{it} is the firm's 3-digit industry sales growth. SG_{it} is the firm's sales growth.

$$
SA = -0.737 \cdot SIZE_{it} + 0.043 \cdot SIZE_{it}^{2} - 0.040 \cdot AGE_{it}
$$

where $SIZE_{it}$ is the log of inflation-adjusted book assets. AGE_{it} is the number of years the firm is listed with a non-missing stock price on Compustat. The following formula is our financial constraint index created by principal component analysis

$$
FC - Index = \alpha_1 KZ + \alpha_2 WW + \alpha_3 SA
$$

The advantage of our financial constraint index is that it considers the multi-dimension of a firm's financial health. In our empirical result, the first principal component can explain about 70% to 80% of the sample variance (unreported). As a result, we take the weight of factors of the first principal component as the parameter estimates of the new financial constraint index.

IV. Data Description

A. Default Data Set

The default data selection refers to Duffie et al. (2007) and Duan et al. (2011a). Our sample period is from 1991 to 2009. The accounting data is from Compustat quarterly file. The stock market information is from CRSP monthly and daily files. We only chose companies that trade on NYSE, AMEX and NASDAQ (exchange code 1 to 3) with share type code 10 and 11 (common stocks). The exit types of firms are from CRSP and Compustat database. Firms are regarded as bankruptcy if the CRSP delisting code is "574" and the delisting reason is "02" and "03" in Compustat. We categorize the firms into merge group if the delisting reason is "01"; the others are placed in the other-exit group. If the accounting data is missing, we complete the data by Compustat yearly file or closest quarterly data prior to the missing data date. Finally, we have 3,412 firms (exclude financial firms) and 273,881 firm-month observations in our data set.

 In order to estimate the default probability, in addition to the market covariates, we add some firm-specific covariates as our explanatory variables. Even the timing that information of accounting covariates is announced is lagged behind the market covariates, there is some internal information that is unknown to the investor. Shumway (2001) and Duan et al. (2011a) add the accounting covariates to improve the accuracy of prediction. The following are the covariates we use:

- 1. SP500: the trailing one-year return on the S&P 500 index.
- 2. TB3M: 3-month US Treasury bill rate.
- 3. Return: trailing one-year stock return of each firm.
- 4. DTD: firm's distance to default.
- 5. CASHTA: ratio of cash and short-term investment to total assets.
- 6. NITA: ratio of net income to total assets.
- 7. SIZE: log of the ratio firm's market value to the average market value of the S&P 500

firm.

 We considered the trend of covariates, and calculated the average value of the covariates over the preceding the 12-month moving average. And we also considered the incremental value of the covariates, calculated the difference between the current value and the 12-month moving average value. We denote it by the subscript "AVG" and "DIF" to represent the average value and incremental value of covariates. To eliminate the influence of the outlier, we winsorize the firm-specific covariates at 1st and 99th percentile value. Table I shows the number of firm types in our sample period from the Compustat database.

B. VAR State Variables

In order to calculate the cash-flow beta and discount-rate beta, we have to estimate the cash-flow news and discount-rate news respectively. We follow Campbell and Vuolteenaho (2004) VAR approach, which used the following four state variables. The first state variable is the excess log market return (*Rm*), which is the difference between the log CRSP value-weighted index and the log risk-free rate. The risk-free rate is the 3-month US Treasury bill rate. The second state variable is the default spread (*DEF*), which is the yield difference between Moody's BAA and AAA bonds. The *DEF* is from the website of the Federal Reserve Bank of St. Louis. Third, price to earnings ratio (*PE*) is from Shiller (2000). The PE ratio is calculated by the price of the S&P 500 index divided by a ten-year moving average earnings of the total S&P 500 firms. The last state variable is the small-stock value spread (*VS*), while the following statement is the construction process of *VS*. At the end of June of year t, we calculate *VS* as the difference between the log(BM) of small high BM portfolio and the log(BM) of small low BM portfolio. The *VS* of July is constructed by the *VS* of June adding the cumulative log return on the small low BM portfolio and subtracting the cumulative log return on the small high BM portfolio from June. We calculate the following months similarly until the next May.

 Since the series of 10-Year constant maturity rate and 3-Month Treasury Bill rate start from 1953, we didn't use the term yield spread (*TY*) from Campbell, Gihlio, and Polk (2010a), calculated as the difference between log yield of 10-Year constant maturity rate and the log yield 3-Month Treasury Bill. Campbell et al. (2010a) show that the estimation results of two-variable (*Rm* and *PE*) and five-variable VAR are robust.

V. Empirical Results

A. Summary Statistics

Table II shows the descriptive statistics of the firm-specific covariates of the default data set. Panel A in Table I describes the summary statistics of the entire sample. We divide the entire sample into two groups: bankrupted group and survival group. The summary statistics of two groups are reported in Panel B and Panel C respectively. It is obvious that the mean of all covariates of bankruptcy group is smaller than the survival group. The distance to default of bankruptcy firms is relatively small, it shows the probability that they will bankrupt in the future. It's not surprising that bankruptcy firms have negative returns past one year (the mean is -2.616% and the median is -4.717%). Cash and short term investment of bankruptcy firms are somewhat lower, but the gap between two groups is not wide (the median of bankruptcy and survival group are 0.026 and 0.056 respectively). The ratio of net income, representation of firm's profitability, is 4 times smaller at the median for bankruptcy group. Almost 50% of firms have net negative incomes (the median is 0.003). The firm size of bankruptcy is about 5 times smaller at the median relative to survival group.

B. Results of Parameter Estimation

VAR Model

Table III reports the VAR parameter estimates. Our result is consistent with Campbell and Vuolteenaho (2004) and Campbell et al. (2010a). The coefficient of lagged excess return in the first row is 0.095; it positively predicts the future market return. The small value spread is positive and insignificant. Campbell et al. (2010a) indicate that the small value and default spread are highly correlated; it is the reason that caused the sign of small value spread is different. We find the correlation coefficient between small value spread and default spread is 0.500. The remain state variables have the $AR(1)$ process, where the coefficient of their lagged term is close to 1. In the third row, we find the lagged default spread can predict the small value spread, but the small value spread can't predict the default spread.

Default Intensity Model

We present the result of parameter estimation in Table IV. Model 1 and Model 2 are the estimation result using the covariates from Duffie et al. (2007) and Duan et al. (2011a). We find that the result of Model 1 is consistent with Duffie et al. (2007). Distance to default is the most important covariate in the model; there is a negative relationship with the default probability. When firms have lower distance to default and trailing 1-year stock return, they are more likely to suffer bankruptcy. Firms bankrupt more frequently at the economy recession, but trailing 1-year return of S&P 500 is the opposite sign. Duan et al. (2011a) indicate that S&P 500 trailing return is regarded as a correction when a firm's distance to default is overestimated at the economy peak.

 Models 2 to 5 are our estimation results; we consider all covariates in the Model 2. Estimation result of Model 2 is consistent with Duan et al. (2011a), but some covariates they advocate are insignificant. For example, the average value of ratio of cash to total asset and ratio of net income to total asset are insignificant. Our explanation is that cash and net income is highly correlated that makes them influence each other. We find that firm size is another

important prediction factor; the sign is consistent with the literature (Shumway (2001), Campbell et al. (2008), Duan et al. (2011a)). It's well known that small firms are more possibly bankrupted than large firms. In addition, we add the difference between current value and the average value of past 12 months to capture the trend. We find that SIZE_{DEF} is also negative and significant; it suggests that the default possibility declines when firm size becomes larger. Because of the Pseudo- R^2 of Model 5 is the highest, we choose the parsimonious model (Model 5) to compute the default probability of each firm in this paper.

C. Default Risk, Betas, and Stock Returns

Default Risk and Average Returns

We calculate the default probability and financial constraint index of each firm, and divide our sample into five groups based on default probability and financial constraint index respectively. Panel A in table V shows the average excess returns of five groups regressed from the CAPM, the three-factor model of Fama-French (1993), and the four-factor model of Carhart (1997). Panel B reports the factor loadings of the four-factor model. Standard errors are in parentheses. At the left side of Panel A, the average excess returns of default-risk-sorted portfolios are significantly decreased when default risk is increased. The highest default risk portfolio shows the negative return at -0.755% per month (-9.06% per annum). We create a portfolio, DP-Diff., which long the low default risk portfolio and short the high default risk portfolio. The DP-Diff. portfolio has the positively average return of 1.360% per month (16.32% per annum). The remaining rows are the alphas estimated from different asset pricing models. We notice that the pattern of alpha decreases monotonly as default risk rises. We find the alphas of different asset pricing modal are significant in the portfolios with higher default risk after we control the pricing factors. The DP-Diff. portfolio has a CAPM alpha of 1.595 with standard error of 0.551 and it has three-factor alpha of 1.318 with standard error of 0.548. When we use the four-factor model to control the momentum effect, the alphas of different portfolios are still significantly positive.

 At the right side of table V, the average excess returns of FC-Index-sorted portfolios are declining when the financial constraint is worse. We also create a portfolio, FC-Diff., which is long the portfolio of first quintile of financial constraint index and short the portfolio of fifth quintile of financial constraint index. The FC-Diff. has the significantly positive excess return of 1.410% per month (16.920% per annum). We also show the estimated alphas from regression of the excess return on the different asset pricing models. We find that the alphas in the fourth and fifth portfolios are significantly negative after controlling the different pricing factors.

 Now we create the 25 financial constraint index- and default probability-sorted portfolios in table V, and report the characteristics of cash-flow, discount-rate beta and average return of 25 portfolios. The portfolios are exhibited in a 5×5 matrix with default probability from low to high at the horizontal axis, and the level of financial constraint from low to high at the vertical axis. We report the difference between high default probability and low default probability in each financial constraint index portfolio at the right column. And we report the difference between the high level of financial constraint index and the low level of financial constraint index in each default probability group at the bottom row.

 After controlling for the financial constraint effect, the cash-flow and discount-rate beta are increased simultaneously from low default probability to high default probability. We find that the patterns of cash-flow beta and discount-rate beta display a humped shape. Garlappi and Yan (2011) suggest that the equity beta is not increased monotonely; it displays a humped shape when the default risk rises. The patterns of average returns in Panel C are the same with cash-flow and discount-rate beta. The difference of two betas between low default probability and high default probability are significantly positive. The difference of average returns between low default probability and high default probability are extremely negative; it is between -9.911 to -16.221 percent annually. Campbell and Vuolteenaho (2004) suggest that the cash-flow beta is the driver that influences the stock return. In Table VI, we find the cash-flow beta has a small proportion of total beta; it can explain the reason why stocks with higher default risks can expect lower returns.

The patterns of cash-flow and discount-rate beta from low financial constraint to high financial constraint are not obvious after we control the distress effect. But the average returns are obviously decreased, and the range of average returns is between -6.836 to -10.331 percent annually.

Estimation Result of Risk Premium

We estimate the risk premium of cash-flow and discount-rate betas with a cross-sectional regression.

$$
\mathcal{R}_i^e = g_1 \hat{\beta}_{i,CF} + g_2 \hat{\beta}_{i,DR} + e_i
$$

where *i* is the number of our portfolios, and $\mathcal{R}_i^e \equiv \mathcal{R}_i - \mathcal{R}_{rf}$ represents the average excess return on portfolio *i*. We calculate the time-series mean of the cross-sectional coefficients and standard error by using the methodology from Fama and MacBeth (1973), and draw the time-series coefficients of risk premium in figure 1. From figure 1, we find that the volatility of cash-flow beta is higher than the discount-rate beta. It is obviously that the price risk of cash flow beta in our empirical result is higher than the discount-rate beta. The risk premium of cash-flow beta is 2.21% per month (26.50% per annum) with a t-statistic of 1.179, and -0.61% per month (-7.37% per annum) with a t-statistic of 0.651 for discount-rate beta. It is consistent with Campbell and Vuolteenaho (2004), who suggest that the cash-flow beta has a higher price of risk than the discount-rate beta.

Prediction of Cash-Flow and Discount-Rate Betas

Now we want to know whether default risk or financial constraint risk affects cash-flow and discount-rate betas after we know the stock return is influenced by betas. We have residuals from regressions of two betas on the past six-month market return $(R_{M,-6})$, the log dividend yield on the value-weighted NYSE index (DY) , the yield spread between ten-year and one-year T-bonds ($TERM$) from Lewellen and Nagel (2006) and two price factor, SMB and HML from Fama and French (1993). The following is the first stage regression:

 $Beta_{i,j} = \alpha_i + \beta_{1,i,j} R_{M,-6} + \beta_{2,i,j} DY + \beta_{3,i,j} TERM + \beta_{4,i,j} SMB + \beta_{5,ij} HML + e_{i,j}$

where $i = \text{cash-flow}$ and discount-rate, denotes the cash-flow and discount-rate beta respectively and *j* is the number of portfolio. Following Fama and MacBeth (1973), we report the time-series mean and standard error of the cross-sectional coefficients from regressions of residuals on the default probability and financial constraint index

$$
e_{i,j} = \gamma_0 + \gamma_1 Default Probability_j + \gamma_2 FC - Index_j + \varepsilon_j.
$$

After we have the time-series coefficients of default probability and financial constraint index, we report the summary statistics of Variables Predicting Two Betas in Table VII, where CF_{DP} is the coefficient of default probability predicting cash-flow beta and DR_{DP} is the coefficient of default probability predicting discount-rate beta. We find that CF_{DP} and DR_{DP} are -0.079 with a *t*-statistic 0.963 and 0.378 with a *t*-statistic 1.869 respectively. It shows that the cash-flow beta decreases and discount-rate beta increase when default risk rises, and causes the stock return decreases. The coefficients on two betas of the financial constraint index are extremely small, and *t*-statistics are insignificant. As a result, we know that the default risk is one of several factors that can affect two betas, and make the stock return decrease.

VI. Conclusion

Recent studies suggest that the firms with higher default risk earn the anomalously lower

returns (Dichev (1998), Campbell et al. (2008), Garlappi and Yan (2011)). This violates the concept of CAPM introduced from Sharpe (1964) and Lintner (1965). Another issue we are interested in is the financial constraint risk, we believe default risk and financial constraint risk interact themselves. In this paper, we were concerned about the default risk anomaly from the viewpoint of the asset pricing. We exploit the VAR approach from Campbell and Vuolteenaho (2004), and then decompose the traditional CAPM beta into two components: cash-flow beta and discount-rate beta. Following Duffie et al. (2007), we estimate the default intensity model and improve the model by adding the covariates from Duan et al. (2011a). To estimate the financially constrained level of firms, we create a financial constraint index by principal component approach that is composed of KZ, WW, and SA index.

We create 25 financial constraint index- and default probability-sorted portfolios. We find that cash-flow beta and discount-rate beta increases simultaneously as default risk rises, and the pattern of portfolio returns is opposite to two betas. The patterns of Cash-flow beta and discount-rate beta display a humped shape that is consistent with Garlappi and Yan (2011), who suggest a hump-shaped relationship between equity beta and default risk. We find that the stocks with higher default risk have large CAPM beta which is attributed from discount-rate beta, so they earn low returns. Following Fama and MacBeth (1973), we estimate the risk premium of the cash-flow beta and discount-rate beta. The risk premium of cash-flow is positive and that of discount-rate beta is negative but not statistically significant. It is consistent with Campbell and Vuolteenaho (2004), who suggest that cash-flow beta has the higher price of risk than that of discount-rate beta.

Then we investigate whether default risk or financial constraint risk can affect cash-flow beta and discount-rate beta. We control the past six-month market return, log dividend yield, and yield spread from Lewellen and Nagel (2006). Besides, we control the common risk factor, *SMB* and *HML* from Fama and French (1993). We find that the cash-flow beta drops

and discount-rate beta increases as the default risk rises. We hope this paper provides some evidence from the viewpoint of asset pricing to explain the default anomaly.

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Table I.

Firm number of different types in the sample

This table reports the total firm number of different type from 1991 to 2009. We classify firms to these groups by Compustat data item AFTNT35, the reasons for deletion of firms. The delisting code 1 is merger-acquisition, code 2 and 3 is bankruptcy. We regard the firm as other exit when the delisting code neither 1 nor 2 and 3.

Table II

Summery Statistics of Covariates Predicting Default Probability

The table reports the descriptive statistics of the firm-specific and market covariates. DTD is the distance to default. RETURN is the trailing one year return of firms, which is in terms of percentage. CASHTA is the cash and short term investment over the total assets. NITA is the net income over the total assets. SIZE is log of the ratio of firm's market value to the average market value of the S&P 500 firm. The subscript AVG denote the average in the previous 12 month, DIF denote the difference between the current value and AVG value.

Table III.

VAR parameter estimates

The table reports results from first-order VAR model and the correlation matrix. Panel A reports the VAR parameter estimates. First column is the four dependent variables, and remain five columns are the coefficients of the constant term, the log excess market return (r_M^e) , the price to earnings ratio (*PE*), the small-stock value spread (*VS*), the default yield spread (*DEF*). Panel B reports the correlation matrix of the parameters .Standard errors are in parentheses.

Panel A.VAR parameter estimates										
	Constant	$r_{M,t}^e$	PE_t	VS_t	DEF	R-square %	F-Statistic			
$r_{M,t+1}^e$	0.076	0.095	-0.023	0.003	-0.010	2.96	7.46			
	(0.020)	(0.032)	(0.006)	(0.005)	(0.003)					
PE_{t+1}	0.031	0.513	0.989	0.002	-0.004	99.08	2.64×10^{4}			
	(0.014)	(0.021)	(0.004)	(0.003)	(0.002)					
VS_{t+1}	0.006	0.005	0.004	0.982	0.010	98.20	1.34×10^{4}			
	(0.019)	(0.030)	(0.005)	(0.005)	(0.003)					
DEF_{t+1}	0.005	-0.027	0.004	0.009	0.971	95.14	4.79×10^{3}			
	(0.006)	(0.095)	(0.017)	(0.015)	(0.010)					
				Panel B. Correlation Matrix						
Correlation	$r_{M,t+1}^e$	PE_{t+1}	VS_{t+1}	DEF_{t+1}						
$r_{M,t+1}^e$	1.000	-0.006	0.004	-0.132						
		(0.855)	(0.908)	(0.000)						
PE_{t+1}	-0.006	1.000	-0.312	-0.577						
	(0.855)		(0.000)	(0.000)						
VS_{t+1}	0.004	-0.312	1.000	0.500						
	(0.908)	(0.000)		(0.000)						
DEF_{t+1}	-0.132	-0.577	0.500	1.000						
	(0.000)	(0.000)	(0.000)							

Table IV

Estimation Results of Parameter Predicting Default Probability

The table reports results from double-stochastic model of Duffie et al.(2007). Model 1 is the estimation result used the covariates from Duffie et al.(2007). Model 2-5 is the result of our estimation. Standard error is reported in parentheses. *denotes significance at 10%, **denotes significance at 5%, ***denotes significance at 1%.

Table V Portfolio Returns of The Risk Factors

We sort our sample by the distress risk and financial constraint risk. All stocks are divided into five portfolios by the quintile of two risk factors. In Panel A, we show the portfolio alphas regressed from the CAPM, three factors (RM, SMB, HML) and four factors (RM, SMB, HML, UMD) Fama-French factor regressions. The portfolio alphas is monthly and in terms of percentage. Panel B shows the factor loadings on the four factors model. Diff. is the difference between the low risk portfolio and high risk portfolio. Standard errors are in parentheses.

Table VI

The Sorting Results of Cash-Flow and Discount-Rate Betas

The table shows the sorting result of cash-flow and discount-rate betas via default probability and financial constraint index. The horizontal axis is default probability from low to high, and the vertical is financial constraint index from low to high. Panel A is the cash-flow and discount-rate betas estimated using the VAR model from John Campbell and Vuolteenaho (2004). Panel B is the equal-weighted average returns, which are annualized and in terms of percentage. Standard errors are in parentheses.

$\hat{\beta}_{CF}$		Default Prob.-Low		\overline{c}		3	$\overline{4}$		Default Prob.-High		Diff.	
FC-Low	0.128	(0.009)		0.127 (0.010)	0.118	(0.010)	0.126	(0.011)	0.147	(0.012)	0.020	(0.010)
2	0.101	(0.009)	0.180	(0.011)	0.150	(0.011)	0.115	(0.009)	0.118	(0.012)	0.017	(0.008)
3	0.132	(0.008)	0.177	(0.010)	0.140	(0.011)	0.086	(0.010)	0.100	(0.010)	-0.032	(0.006)
4	0.099	(0.009)	0.163	(0.011)	0.080	(0.010)	0.111	(0.008)	0.156	(0.011)	0.057	(0.007)
FC-High	0.105	(0.007)	0.111	(0.008)	0.144	(0.008)	0.156	(0.007)	0.150	(0.011)	0.045	(0.008)
Diff.	-0.022	(0.007)	-0.016	(0.004)	0.026	(0.006)	0.031	(0.006)	0.003	(0.013)		
$\hat{\beta}_{DR}$		Default Prob.-Low	$\overline{2}$		3		$\overline{4}$		Default Prob.-High		Diff.	
FC-Low	0.863	(0.017)	1.087	(0.033)	1.052	(0.022)	1.002	(0.021)	1.075	(0.026)	0.212	(0.024)
2	0.887	(0.016)	0.966	(0.021)	1.051	(0.026)	0.974	(0.018)	0.946	(0.024)	0.059	(0.019)
3	0.845	(0.015)	0.914	(0.018)	0.980	(0.021)	0.926	(0.026)	1.010	(0.031)	0.165	(0.018)
4	0.845	(0.016)	0.967	(0.025)	0.932	(0.024)	0.935	(0.018)	0.972	(0.023)	0.128	(0.011)
FC-High	0.804	(0.011)	1.041	(0.025)	1.019	(0.023)	0.958	(0.018)	1.119	(0.028)	0.315	(0.025)
Diff.	-0.060	(0.014)	-0.046	(0.012)	-0.033	(0.011)	-0.044	(0.012)	0.043	(0.021)		
Panel B. Cross-Sectional Average Equal-Weighted Returns												
$\widehat{\mathrm{E}}(R)$		Default Prob.-Low		$\sqrt{2}$		3		$\overline{4}$		Default Prob.-High	Diff.	
FC-Low	23.422	(5.820)		26.750 (7.320)		18.076 (7.152)	23.537	(6.804)	8.280	(8.940)		$-15.142(6.888)$
$\overline{2}$	17.347	(5.688)	20.895	(7.344)	17.198	(7.116)	16.552	(6.612)	1.603	(8.460)		$-15.744(6.312)$
3	19.556	(5.316)	18.365	(6.396)	19.003	(6.828)	18.055	(5.940)	9.004	(8.052)		$-10.552(5.724)$
4	15.098	(5.232)	17.789	(6.636)	18.195	(6.732)	20.531	(6.384)	5.187	(8.616)		$-9.911(6.420)$
FC-High	14.170	(4.848)	15.136	(6.480)	11.240	(6.300)	14.119	(6.828)	-2.051	(9.036)	-16.221	(6.768)
Diff.	-9.252	(3.708)		-11.614 (4.020)	-6.836	(4.008)	-9.418		$(4.188) -10.331$	(5.856)		

Panel A. Cash-Flow and Discount-Rate Betas

Table VII

Summary Statistics of Time-Series Coefficient Predicting Two Betas

Following Fama and MacBath (1973), we report the summary statistics of default probability and financial constraint index's time-series coefficients. DP_CF is the coefficient of default probability predicting cash-flow beta. FC_CF is the coefficient of financial constraint index predicting cash-flow beta. DP_CF is the coefficient of default probability predicting discount-rate beta. FC_CF is the coefficient of financial constraint index predicting discount-rate beta.

