東海管理評論
民國九十年七月，第三卷，第一期，155-172

Power Comparison of Tests for a Common Mean of Several Gamma Populations Based on P-values

Wei-Hsiung Shen Yih-Jy Pan***

Abstract

In this note we provide a power comparison of several test procedures for the common mean of k Gamma populations. These tests are based on well known combinations of so-called P-values, such as Tippett / Wilkinson, Fisher / Good, Inverse Normal and Logit method.

Keywords：Fisher's P-values, Meta analysis, Power.

1. Introduction

Meta-analysis deals with the discipline of combining findings from repeated research studies all of which are targeted to address a common goal. Some of applications are in agriculture, aviation, educational and psychological research, effectiveness of pollution prevention program, medicine and clinical experiments, biology, physics, and social sciences.

Assume the experiment E_i results in data from $f(x;\theta,\delta_i)$, $i=1,\ldots,k$, where $\bar{\theta}$ is the vector of common parameters of interest and $\delta_1,\ldots,\delta_k$ are the vectors of nuisance parameters. The problem is to make inference on θ by combining results of k experiments.

The comparison of powers of tests for the "common normal mean"

* Professor of Department of Statistics, Tunghai University
** Business and Technology Integration Department, Chinatrust Commercial Bank

problem has been addressed by Lin and Shen (1996) in which $f(x;\theta,\delta_i)$ was assumed to be $N(\mu,\sigma_i^2)$, where μ is the common mean and $\sigma_1^2,\ldots,\sigma_k^2$ are the unknown variances. Later on, Lin and Shen (1997) also compared the powers of tests for the common location when $f(x;\theta,\delta_i)$ is truncated exponential.

In this note we assume $f(x;\theta,\delta_i)$ to be $Gamma(\theta_i,\beta_i)$ with the probability density function

$$f(x)=\frac{1}{\Gamma(\theta_i)}\beta_i^{\theta_i}x^{\theta_i-1}e^{-\beta_i x} \qquad \theta_i>0,\ \ \beta_i>0,\ \ i=1,\ldots,k.$$

We assume that the shape parameters θ_i, $i=1,\ldots,k$ are known, the scale parameters β_i's are unknown, and also that the means are all equal, i.e., $(\theta_1/\beta_1)=(\theta_2/\beta_2)=\ldots=(\theta_k/\beta_k)=\eta$ (say). Under this set up, we want to test $H_0:\eta=\eta_0$ against $H_1:\eta>\eta_0$, where η_0 is a known constant. One can also consider the situation when the scale parameters β_i's are known, the shape parameters θ_i's are unknown, and the testing problem as above. However, the latter formulation is quite difficult, and we do not discuss it here.

For each experiment, base on $x_{i1},x_{i2},\ldots,x_{in_i}$ from E_i, the likelihood ratio test statistic is based on

$$\Lambda_i=\bar{x}_i=\frac{1}{n_i}\sum_{j=1}^{n_i}x_{ij} \qquad i=1,\ldots,k.$$

and Λ_i is distributed as $Gamma(n_i\theta_i,n_i\theta_i/\eta_0)$ under H_0 where n_i is number of observations from the ith experiment. Fisher (1932) recommended to combine these independent tests based on their P-values.

In Section 2, we summarize the commonly used procedures of combining

P-values, such as Tippett / Wilkinson, Fisher / Good, Inverse Normal and Logit methods. In Section **3**, we provide power comparisons of all those test procedures for testing H_0 and for different number k of experiments and sample sizes, respectively.

2. Tests Based on P-Values

We first begin by recalling the definition of the P-value. Suppose $\Lambda_i(X_{i1}, X_{i2}, \ldots, X_{in_i})$ is a continuous test statistic for H_0 from experiment E_i and λ_i is the observed value of Λ_i, $i = 1, \ldots, k$. The P-value based on data from E_i can be computed as

$$\boldsymbol{P_i \equiv \Pr[\Lambda_i > \lambda_i \mid H_0]} \quad i = 1, \ldots, k.$$

We reject the null hypothesis if P-value is small. Note that the P-value is distributed as Uniform on $[0,1]$ under the null hypothesis.

We summarize below some standard methods for combining these independent P-values.

2.1. Tippett／Wilkinson method

Tippett (1931) suggested using the smallest P_i, i.e., $P_{[1]} \equiv \min_i(P_i)$, and reject H_0 if

$$\boldsymbol{P_{[1]} < 1 - (1-\alpha)^{1/k}} \quad i = 1, \ldots, k.$$

where α is the significance level. Its power at $\eta > \eta_0$ is obtained as

$$\boldsymbol{Power\ (\eta \mid P_{[1]}) = \Pr\ \{P_{[1]} < 1 - (1-\alpha)^{1/k} \mid \eta\}} \quad i = 1, \ldots, k.$$

Wilkinson (1951) generalized Tippett's method by using the r th smallest P_i, i.e., $P_{[r]} \equiv r$ th smallest P-value and reject H_0 if

$$\boldsymbol{P_{[r]} < c}$$

and with its power at $\eta > \eta_0$ as

$$\boldsymbol{Power\ (\eta \mid P_{[r]}) = \mathrm{Pr}\ \{P_{[r]} < c \mid \eta\}}$$

where c is the upper α level cut-off point. Note that $P_{[r]}$ is distributed as $Beta[r, k-r+1]$ under H_0.

2.2. Fisher／Good method

Since P_i is distributed as $U(0,1)$ under H_0, then $-2 \log P_i$ is distributed as chi-squared with 2 $d.f.$. Hence, Fisher(1932)proposed to use the sum of these k independent chi-squared statistics. i.e., $P_F = -2\sum_1^k \log P_i$ and reject H_0 if

$$\boldsymbol{P_F > \chi^2_{2k;\alpha}}$$

and with its power at $\eta > \eta_0$ as

$$\boldsymbol{Power\ (\eta \mid P_F) = \mathrm{Pr}\ \{P_F > \chi^2_{2k;\alpha} \mid \eta\}}.$$

Good (1955) modified Fisher's method by assigning greater weights to more sensitive studies. Define $P_W = \prod_1^k P_i^{w_i}$ where w_i is the weight of i th experiment. The asymptotic distribution of $-2 \log P_W$ is $\tau\chi^2_\nu$ where $\tau = \sum_1^k w_i^2 / \sum_1^k w_i$ and $\nu = 2\left(\sum_1^k w_i\right)^2 / \sum_1^k w_i^2$. Hence, we define $P_G = -2 \log P_W / \tau$ and reject H_0 if

$$P_G > \chi^2_{v;\alpha}$$

and with its power at $\eta > \eta_0$ as

$$Power\,(\eta \mid P_G) = \Pr\{P_G > \chi^2_{v;\alpha} \mid \eta\}$$

2.3. Inverse Normal method

If we transform each P-value to a Normal score, i.e., $Z = \Phi^{-1}(P)$, then Z is distributed as standard normal under H_0.

Stouffer, Suchman, DeVinney, Star and Williams (1949) and Liptak (1958) proposed that we reject H_0 if

$$P_Z = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} Z_i < -\xi_\alpha$$

and with its power at $\eta > \eta_0$ as

$$Power\,(\eta \mid P_Z) = \Pr\{P_Z < -\xi_\alpha \mid \eta\}$$

where ξ_α is the upper α level cut-off point of $N(0,1)$.

2.4. Logit method

George(1977)and George and Mudholker(1977)suggested to transform each P-value to $log[P/(1-P)]$ and define $L = \sum_1^k log[P_i/(1-P_i)]$. However, under H_0, it is very difficult to find the exact distribution of L. Hence, we may use the approximate test procedure that rejects H_0 if

$$P_L = L\sqrt{\frac{0.3(5k+4)}{k(5k+2)}} < -t_{\alpha;(5k+4)}$$

and with its power at $\eta > \eta_0$ as

$$\boldsymbol{Power}\ (\eta \mid P_L) = \Pr\{P_L < -t_{\alpha;(5k+4)} \mid \eta\}$$

where $t_{\alpha;(5k+4)}$ is the upper α level cut-off point of t distribution with $(5k+4)$ $d.f.$.

3. Comparison of Powers

In this section we provide a power comparison for all the test procedures described in Section **2**.

Without loss of generality we take $\eta_0 = 1$ for H_0. In each case of $k = 2, 3$, and η=1.2, 1.4, 1.6, 1.8 and 2.0, the simulation was replicated 10,000 times, and the significance level is taken as $\alpha = 0.05$.

For Wilkinson method, the largest P-value of k experiments was used in this study. The weights used in Good's method are $w_i = (n_i \sqrt{\theta_i}) / (\sum_{i=1}^{k} n_i \sqrt{\theta_i})$.

For k=2, we choose θ_1=1 and θ_2=1.5, 2.0, 3.0, 4.0 and 5.0. When the sample sizes are equal, 《Tab.1》, 《Tab.2》 and 《Tab.3》 show the powers of the test procedures for 2 experiments, each having sample sizes $n_1 = n_2$=5, 10, 15. For the unequal sample sizes, 《Tab.4》, 《Tab.5》 and 《Tab.6》 show the powers of the test procedures for different combinations of sample sizes of n=5, 10, 15.

For k=3, we took only equal sample sizes $n_1 = n_2 = n_3$=5, 10, 15 and chose the combinations (θ_1, θ_2, θ_3) = (1.0, 1.5, 2.0) , (1.0, 2.0, 3.0) and (1.0, 5.0, 10) . 《Tab.7》, 《Tab.8》 and 《Tab.9》 show the powers of the test procedures.

Our finding can be summarized as follows. In general, Fisher's method performs the best, then Inverse Normal and Logit method. Not surprisingly, Tippett and Wilkinson methods perform poorly because Tippett method uses the smallest P-value and we use the largest P-value for Wilkinson method in this note, Good's method also does not perform well perhaps due to the choice of weights.

Interestingly enough, our overall findings for the gamma case coincide with those under the normal case reported in Lin and Shen (1996).

《Tab.1》 Powers of tests based on P-values, $k=2$, $n_1=n_2=5$

θ	η	Power					
		Tippett	***Wilkinson***	***Fisher***	***Good***	***Inv Nor***	***Logit***
θ_1=1.0 θ_2=1.5	1.2	0.1523	0.1507	0.1730	0.1669	0.1740	0.1729
	1.4	0.3033	0.2840	0.3468	0.3397	0.3485	0.3496
	1.6	0.4765	0.4376	0.5416	0.5322	0.5422	0.5476
	1.8	0.6134	0.5551	0.6859	0.6785	0.6845	0.6888
	2.0	0.7386	0.6684	0.7988	0.7929	0.7988	0.8027
θ_1=1.0 θ_2=2.0	1.2	0.1687	0.1538	0.1873	0.1776	0.1832	0.1867
	1.4	0.3435	0.3137	0.3925	0.3732	0.3906	0.3941
	1.6	0.5298	0.4745	0.5941	0.5676	0.5924	0.5982
	1.8	0.6799	0.6014	0.7475	0.7252	0.7433	0.7517
	2.0	0.7889	0.7001	0.8471	0.8298	0.8441	0.8477
θ_1=1.0 θ_2=3.0	1.2	0.1936	0.1804	0.2176	0.1887	0.2198	0.2192
	1.4	0.3984	0.3530	0.4574	0.4017	0.4534	0.4582
	1.6	0.6157	0.5266	0.6797	0.6194	0.6727	0.6809
	1.8	0.7826	0.6508	0.8334	0.7805	0.8237	0.8314
	2.0	0.8821	0.7370	0.9161	0.8847	0.9084	0.9157
θ_1=1.0 θ_2=4.0	1.2	0.2171	0.1926	0.2390	0.1930	0.2373	0.2392
	1.4	0.4640	0.3859	0.5192	0.4225	0.5063	0.5143
	1.6	0.6987	0.5547	0.7552	0.6652	0.7402	0.7522
	1.8	0.8503	0.6865	0.8880	0.8268	0.8791	0.8874
	2.0	0.9327	0.7619	0.9537	0.9188	0.9448	0.9509
θ_1=1.0 θ_2=5.0	1.2	0.2232	0.2072	0.2531	0.1964	0.2535	0.2529
	1.4	0.5144	0.4111	0.5711	0.4380	0.5522	0.5630
	1.6	0.7572	0.5687	0.8025	0.6856	0.7832	0.7960
	1.8	0.9024	0.6912	0.9281	0.8578	0.9129	0.9251
	2.0	0.9639	0.7645	0.9751	0.9352	0.9653	0.9726

《Tab.2》 Powers of tests based on P-values, $k=2$, $n_1=n_2=10$

θ	η	Power					
		Tippett	Wilkinson	Fisher	Good	Inv Nor	Logit
$\theta_1=1.0$ $\theta_2=1.5$	1.2	0.2149	0.2111	0.2459	0.2385	0.2476	0.2490
	1.4	0.4564	0.4418	0.5268	0.5175	0.5345	0.5358
	1.6	0.6891	0.6456	0.7607	0.7542	0.7672	0.7674
	1.8	0.8474	0.7841	0.8971	0.8934	0.9001	0.9019
	2.0	0.9260	0.8695	0.9561	0.9539	0.9577	0.9581
$\theta_1=1.0$ $\theta_2=2.0$	1.2	0.2356	0.2285	0.2734	0.2579	0.2731	0.2720
	1.4	0.5101	0.4711	0.5798	0.5545	0.5837	0.5857
	1.6	0.7601	0.6795	0.8237	0.8066	0.8261	0.8299
	1.8	0.8932	0.8119	0.9330	0.9206	0.9307	0.9343
	2.0	0.9586	0.9015	0.9790	0.9754	0.9788	0.9798
$\theta_1=1.0$ $\theta_2=3.0$	1.2	0.2718	0.2646	0.3125	0.2716	0.3186	0.3191
	1.4	0.6118	0.5295	0.6822	0.6121	0.6783	0.6856
	1.6	0.8487	0.7202	0.8964	0.8548	0.8879.	0.8952
	1.8	0.9577	0.8422	0.9735	0.9590	0.9717	0.9742
	2.0	0.9895	0.9089	0.9946	0.9901	0.9925	0.9944
$\theta_1=1.0$ $\theta_2=4.0$	1.2	0.2996	0.2758	0.3454	0.2740	0.3446	0.3486
	1.4	0.6993	0.5645	0.7573	0.6608	0.7491	0.7591
	1.6	0.9087	0.7451	0.9386	0.8882	0.9308	0.9370
	1.8	0.9829	0.8434	0.9898	0.9760	0.9871	0.9891
	2.0	0.9969	0.9088	0.9986	0.9959	0.9980	0.9986
$\theta_1=1.0$ $\theta_2=5.0$	1.2	0.3459	0.3041	0.3989	0.2939	0.3952	0.4003
	1.4	0.7587	0.5850	0.8121	0.6867	0.7931	0.8079
	1.6	0.9516	0.7504	0.9683	0.9233	0.9587	0.9659
	1.8	0.9917	0.8509	0.9958	0.9863	0.9934	0.9952
	2.0	0.9988	0.9080	0.9993	0.9978	0.9988	0.9991

《Tab.3》 Powers of tests based on P-values, $k=2$, $n_1=n_2=15$

θ	η	Power					
		Tippett	Wilkinson	Fisher	Good	Inv Nor	Logit
θ_1=1.0 θ_2=1.5	1.2	0.2632	0.2601	0.3075	0.2999	0.3114	0.3107
	1.4	0.5853	0.5594	0.6719	0.6627	0.6769	0.6777
	1.6	0.8280	0.7864	0.8896	0.8827	0.8937	0.8947
	1.8	0.9416	0.8981	0.9704	0.9669	0.9725	0.9721
	2.0	0.9832	0.9510	0.9932	0.9922	0.9928	0.9932
θ_1=1.0 θ_2=2.0	1.2	0.2915	0.2940	0.3446	0.3276	0.3553	0.3535
	1.4	0.6468	0.6024	0.7264	0.7033	0.7353	0.7366
	1.6	0.8809	0.8173	0.9268	0.9145	0.9262	0.9284
	1.8	0.9681	0.9115	0.9831	0.9800	0.9838	0.9842
	2.0	0.9935	0.9589	0.9973	0.9969	0.9972	0.9973
θ_1=1.0 θ_2=3.0	1.2	0.3528	0.3308	0.4136	0.3604	0.4134	0.4138
	1.4	0.7437	0.6375	0.8084	0.7537	0.8047	0.8116
	1.6	0.9504	0.8384	0.9717	0.9550	0.9686	0.9719
	1.8	0.9914	0.9200	0.9963	0.9929	0.9955	0.9961
	2.0	0.9995	0.9589	0.9997	0.9992	0.9994	0.9997
θ_1=1.0 θ_2=4.0	1.2	0.4016	0.3528	0.4596	0.3677	0.4534	0.4598
	1.4	0.8400	0.6890	0.8886	0.8148	0.8784	0.8885
	1.6	0.9790	0.8445	0.9891	0.9721	0.9846	0.9882
	1.8	0.9981	0.9214	0.9993	0.9974	0.9990	0.9991
	2.0	0.9999	0.9650	1.0000	0.9999	1.0000	1.0000
θ_1=1.0 θ_2=5.0	1.2	0.4605	0.3897	0.5210	0.3919	0.5123	0.5199
	1.4	0.8907	0.6989	0.9227	0.8387	0.9115	0.9190
	1.6	0.9912	0.8508	0.9955	0.9822	0.9912	0.9945
	1.8	0.9997	0.9232	0.9998	0.9992	0.9997	0.9999
	2.0	1.0000	0.9603	1.0000	1.0000	1.0000	1.0000

《Tab.4》 Powers of tests based on P-values, $k=2$, $n_1=5$, $n_2=10$

θ	η	Power					
		Tippett	*Wilkinson*	*Fisher*	*Good*	*Inv Nor*	*Logit*
θ_1=1.0 θ_2=1.5	1.2	0.1864	0.1809	0.2183	0.2211	0.2201	0.2186
	1.4	0.4057	0.3584	0.4585	0.4751	0.4573	0.4596
	1.6	0.6155	0.5252	0.6805	0.6925	0.6735	0.6812
	1.8	0.7790	0.6507	0.8282	0.8385	0.8198	0.8260
	2.0	0.8840	0.7377	0.9192	0.9224	0.9116	0.9183
θ_1=1.0 θ_2=2.0	1.2	0.2173	0.2004	0.2421	0.2533	0.2397	0.2420
	1.4	0.4671	0.3906	0.5218	0.5398	0.5086	0.5203
	1.6	0.6958	0.5558	0.7489	0.7651	0.7343	0.7445
	1.8	0.8559	0.6763	0.8913	0.9027	0.8763	0.8880
	2.0	0.9354	0.7654	0.9567	0.9613	0.9487	0.9554
θ_1=1.0 θ_2=3.0	1.2	0.2508	0.2183	0.2801	0.2897	0.2729	0.2796
	1.4	0.5666	0.4367	0.6191	0.6329	0.6000	0.6128
	1.6	0.8188	0.5787	0.8531	0.8619	0.8249	0.8435
	1.8	0.9323	0.7047	0.9517	0.9547	0.9376	0.9484
	2.0	0.9813	0.7715	0.9870	0.9883	0.9780	0.9846
θ_1=1.0 θ_2=4.0	1.2	0.2898	0.2389	0.3204	0.3204	0.3096	0.3143
	1.4	0.6602	0.4590	0.6984	0.6984	0.6677	0.6877
	1.6	0.8905	0.6094	0.9158	0.9158	0.8852	0.9053
	1.8	0.9744	0.7082	0.9808	0.9808	0.9686	0.9767
	2.0	0.9944	0.7702	0.9965	0.9965	0.9935	0.9955
θ_1=1.0 θ_2=5.0	1.2	0.3270	0.2600	0.3589	0.3463	0.3448	0.3523
	1.4	0.7357	0.4704	0.7682	0.7534	0.7271	0.7498
	1.6	0.9384	0.6132	0.9511	0.9470	0.9264	0.9434
	1.8	0.9909	0.6989	0.9933	0.9925	0.9848	0.9904
	2.0	0.9981	0.7700	0.9985	0.9984	0.9966	0.9984

《Tab.5》 Powers of tests based on P-values, $k=2$, $n_1=5$, $n_2=15$

θ	η	Power					
		Tippett	*Wilkinson*	*Fisher*	*Good*	*Inv Nor*	*Logit*
θ_1=1.0 θ_2=1.5	1.2	0.2194	0.2072	0.2492	0.2612	0.2523	0.2526
	1.4	0.4961	0.4015	0.5471	0.5710	0.5358	0.5473
	1.6	0.7359	0.5730	0.7869	0.7978	0.7651	0.7801
	1.8	0.8781	0.6847	0.9124	0.9211	0.8974	0.9072
	2.0	0.9500	0.7649	0.9667	0.9684	0.9541	0.9626
θ_1=1.0 θ_2=2.0	1.2	0.2482	0.2097	0.2752	0.2990	0.2706	0.2769
	1.4	0.5698	0.4310	0.6213	0.6544	0.5966	0.6109
	1.6	0.8185	0.5913	0.8554	0.8722	0.8313	0.8492
	1.8	0.9335	0.6936	0.9540	0.9627	0.9389	0.9494
	2.0	0.9780	0.7757	0.9858	0.9890	0.9780	0.9840
θ_1=1.0 θ_2=3.0	1.2	0.3111	0.2517	0.3406	0.3718	0.3298	0.3358
	1.4	0.7060	0.4742	0.7475	0.7796	0.7116	0.7363
	1.6	0.9183	0.6061	0.9352	0.9498	0.9063	0.9259
	1.8	0.9822	0.7104	0.9879	0.9907	0.9789	0.9859
	2.0	0.9966	0.7751	0.9983	0.9988	0.9947	0.9969
θ_1=1.0 θ_2=4.0	1.2	0.3706	0.2749	0.4028	0.4358	0.3800	0.3897
	1.4	0.8026	0.4839	0.8288	0.8575	0.7839	0.8110
	1.6	0.9652	0.6110	0.9734	0.9798	0.9475	0.9648
	1.8	0.9959	0.7024	0.9964	0.9978	0.9905	0.9952
	2.0	0.9997	0.7759	0.9999	1.0000	0.9988	0.9998
θ_1=1.0 θ_2=5.0	1.2	0.4238	0.2929	0.4547	0.4846	0.4224	0.4384
	1.4	0.8654	0.4983	0.8789	0.8985	0.8312	0.8594
	1.6	0.9874	0.6173	0.9902	0.9924	0.9753	0.9852
	1.8	0.9984	0.7043	0.9992	0.9992	0.9966	0.9987
	2.0	0.9998	0.7684	0.9999	0.9999	0.9993	0.9998

《Tab.6》 Powers of tests based on P-values, $k=2$, $n_1=10$, $n_2=15$

θ	η	Power					
		Tippett	***Wilkinson***	***Fisher***	***Good***	***Inv Nor***	***Logit***
$\theta_1=1.0$ $\theta_2=1.5$	1.2	0.2413	0.2364	0.2820	0.2866	0.2839	0.2846
	1.4	0.5360	0.4927	0.6120	0.6177	0.6148	0.6167
	1.6	0.7755	0.6903	0.8378	0.8439	0.8371	0.8421
	1.8	0.9125	0.8214	0.9450	0.9471	0.9433	0.9467
	2.0	0.9705	0.8992	0.9840	0.9845	0.9836	0.9850
$\theta_1=1.0$ $\theta_2=2.0$	1.2	0.2709	0.2641	0.3122	0.3158	0.3138	0.3120
	1.4	0.6078	0.5304	0.6758	0.6802	0.6742	0.6796
	1.6	0.8459	0.7249	0.8951	0.8973	0.8888	0.8945
	1.8	0.9554	0.8452	0.9750	0.9756	0.9696	0.9744
	2.0	0.9882	0.9115	0.9951	0.9956	0.9947	0.9954
$\theta_1=1.0$ $\theta_2=3.0$	1.2	0.3377	0.2996	0.3806	0.3697	0.3784	0.3838
	1.4	0.7282	0.5756	0.7782	0.7658	0.7645	0.7778
	1.6	0.9325	0.7556	0.9574	0.9534	0.9499	0.9566
	1.8	0.9898	0.8491	0.9923	0.9915	0.9895	0.9918
	2.0	0.9990	0.9037	0.9994	0.9993	0.9985	0.9992
$\theta_1=1.0$ $\theta_2=4.0$	1.2	0.3856	0.3245	0.4267	0.3942	0.4210	0.4253
	1.4	0.8156	0.5967	0.8592	0.8293	0.8342	0.8513
	1.6	0.9710	0.7509	0.9806	0.9743	0.9713	0.9787
	1.8	0.9975	0.8493	0.9988	0.9984	0.9981	0.9988
	2.0	0.9997	0.9084	1.0000	1.0000	1.0000	1.0000
$\theta_1=1.0$ $\theta_2=5.0$	1.2	0.4479	0.3453	0.4908	0.4301	0.4707	0.4826
	1.4	0.8753	0.6165	0.9027	0.8673	0.8794	0.8957
	1.6	0.9895	0.7616	0.9940	0.9889	0.9881	0.9925
	1.8	0.9993	0.8565	0.9994	0.9992	0.9985	0.9994
	2.0	1.0000	0.9071	1.0000	1.0000	1.0000	1.0000

《Tab.7》 Powers of tests based on P-values, $k=3$, $n_1=n_2=n_3=5$

θ	η	Power					
		Tippett	*Wilkinson*	*Fisher*	*Good*	*Inv Nor*	*Logit*
θ_1=1.0 θ_2=1.5 θ_3=2.0	1.2	0.1838	0.1753	0.2359	0.2268	0.2331	0.2343
	1.4	0.3873	0.3525	0.4976	0.4823	0.5009	0.5039
	1.6	0.6077	0.5234	0.7305	0.7132	0.7281	0.7351
	1.8	0.7583	0.6527	0.8633	0.8520	0.8629	0.8682
	2.0	0.8734	0.7537	0.9422	0.9350	0.9402	0.9448
θ_1=1.0 θ_2=2.0 θ_3=3.0	1.2	0.2053	0.2072	0.2682	0.2421	0.2719	0.2742
	1.4	0.4649	0.4041	0.5855	0.5371	0.5866	0.5932
	1.6	0.7023	0.5885	0.8148	0.7826	0.8148	0.8222
	1.8	0.8534	0.7166	0.9327	0.9129	0.9290	0.9331
	2.0	0.9388	0.8047	0.9747	0.9645	0.9767	0.9782
θ_1=1.0 θ_2=5.0 θ_3=10	1.2	0.3709	0.3122	0.4700	0.2938	0.4645	0.4723
	1.4	0.8017	0.5800	0.8872	0.7289	0.8758	0.8847
	1.6	0.9706	0.7209	0.9898	0.9442	0.9836	0.9879
	1.8	0.9973	0.8103	0.9994	0.9947	0.9989	0.9991
	2.0	1.0000	0.8608	1.0000	0.9997	0.9999	1.0000

《Tab.8》 Powers of tests based on P-values, $k=3$, $n_1=n_2=n_3=10$

θ	η	Power					
		Tippett	*Wilkinson*	*Fisher*	*Good*	*Inv Nor*	*Logit*
θ_1=1.0 θ_2=1.5 θ_3=2.0	1.2	0.2582	0.2518	0.3439	0.3283	0.3495	0.3489
	1.4	0.5755	0.5373	0.7205	0.7014	0.7260	0.7285
	1.6	0.8357	0.7409	0.9293	0.9202	0.9327	0.9352
	1.8	0.9488	0.8677	0.9854	0.9823	0.9864	0.9865
	2.0	0.9867	0.9330	0.9974	0.9969	0.9977	0.9977
θ_1=1.0 θ_2=2.0 θ_3=3.0	1.2	0.3078	0.3069	0.4082	0.3706	0.4181	0.4185
	1.4	0.6880	0.6105	0.8156	0.7766	0.8196	0.8236
	1.6	0.9131	0.7979	0.9720	0.9584	0.9713	0.9734
	1.8	0.9827	0.8955	0.9966	0.9945	0.9967	0.9968
	2.0	0.9970	0.9432	0.9994	0.9991	0.9991	0.9992
θ_1=1.0 θ_2=5.0 θ_3=10	1.2	0.5786	0.4443	0.7068	0.4924	0.6964	0.7057
	1.4	0.9715	0.7288	0.9919	0.9474	0.9881	0.9917
	1.6	0.9999	0.8476	1.0000	0.9987	1.0000	1.0000
	1.8	1.0000	0.9175	1.0000	1.0000	1.0000	1.0000
	2.0	1.0000	0.9533	1.0000	1.0000	1.0000	1.0000

《Tab.9》 Powers of tests based on P-values, $k=3$, $n_1=n_2=n_3=15$

θ	η	Power					
		Tippett	***Wilkinson***	***Fisher***	***Good***	***Inv Nor***	***Logit***
θ_1=1.0 θ_2=1.5 θ_3=2.0	1.2	0.3169	0.3309	0.4357	0.4187	0.4509	0.4483
	1.4	0.7242	0.6659	0.8604	0.8467	0.8698	0.8705
	1.6	0.9329	0.8652	0.9807	0.9777	0.9830	0.9833
	1.8	0.9888	0.9435	0.9980	0.9975	0.9984	0.9985
	2.0	0.9988	0.9786	0.9998	0.9998	1.0000	1.0000
θ_1=1.0 θ_2=2.0 θ_3=3.0	1.2	0.3953	0.3971	0.5329	0.4851	0.5431	0.5443
	1.4	0.8241	0.7387	0.9261	0.9021	0.9295	0.9304
	1.6	0.9786	0.8906	0.9945	0.9915	0.9945	0.9950
	1.8	0.9988	0.9597	1.0000	0.9997	1.0000	1.0000
	2.0	0.9999	0.9817	1.0000	1.0000	1.0000	1.0000
θ_1=1.0 θ_2=5.0 θ_3=10	1.2	0.7326	0.5474	0.8423	0.6484	0.8325	0.8428
	1.4	0.9964	0.8196	0.9996	0.9937	0.9990	0.9995
	1.6	1.0000	0.9152	1.0000	1.0000	1.0000	1.0000
	1.8	1.0000	0.9612	1.0000	1.0000	1.0000	1.0000
	2.0	1.0000	0.9807	1.0000	1.0000	1.0000	1.0000

References

Fisher, R.A. (1932), *Statistical Methods for Research Workers*, 4th edition, London : Oliver and Boyd.

George, E.O. (1977), *Combining Independent One-Sided and Two-Sided Statistical Tests: Some Theory and Applications*, Doctoral dissertation, University of Rochester, Rochester, New York.

______, and G.S. Mudholker (1977), *On the Convolution of Logistic Random Variables*, Unpublished paper, University of Rochester, Rochester, New York.

Good, I.J. (1955), "On the Weighed Combination of Significance Test", *Journal of the Royal Statistical Society*, Series B, 17 : 264~65.

Lin, Y-L., and W-H. Shen (1996), "Comparison of Powers of Tests Based on Fisher's P-values", *Journal of Statistical Research*, 30 : 37~48.

______, and ______ (1997), "Comparison of Power of Tests for a Common Location of k Truncated Exponentials", *Journal of Applied Statistical Science*, 6：265～73.

Liptak, T. (1958), "On the Combination of Independent Tests", *Magyer Tudomanyos Akademia Matematikai Kutato Intezetenek Kozlemenyei*, 3：1971～77.

Stouffer, S.A., E.A. Suchman, L.C. DeVinney, S.A. Star, and R.M. Jr. Williams (1949), "The American Soldier: Adjustment During Army Life", *New Jersey: Princeton University Press*, Vol. I., Princeton.

Tippett, L.H.C. (1931), *The Methods of Statistics, London: Williams and Norgate*.

Wilkinson, B. (1951), "A Statistical Consideration in Psychological Research", *Psychological Bulletin*, 48：156～58.

多個伽瑪分配平均數均齊性檢定之檢定力比較

沈維雄* 潘益之**

摘要

本文是想利用**統計整合分析**的方法，對 k (≥ 2)個伽瑪分配平均數的均齊性檢定提出數種檢定方法。我們將討論幾個建立於**費雪P值**上所常用的檢定式，例如 Tippett / Wilkison，Fisher / Good，Inverse Normal 和 Logit 方法。再利用模擬的方法比較這些方法的**檢定力**。

關鍵詞：統計整合分析、費雪P值、檢定力。

* 東海大學統計學系教授

** 中國信託商業銀行 業務資訊整合部