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Tests for Normal Parameters Based on a Ranked Set Sample

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Abstract

In this paper we explore the concept of **ranked set sample** introduced by McIntyre (1952) for the problem of tests for a normal mean and a normal variance, and show that many improved tests can be constructed, all of which are much better than the traditional t test and chi-square test, respectively.

Keywords：Order Statistics, Ranked Set Sample, Simple Random Sample.

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1. Introduction

In sampling situation when the variable of interest to be observed from the experimental units can be more easily ranked than quantified, McIntyre (1952) observed that, for estimation of the population mean, the sample mean based on what he introduced as a 'Ranked Set Sample' (RSS) is unbiased and much superior to that based on a standard simple random sample (SRS). For applications where such rankings can be easily done, we refer to Cobby *et al.* (1985), Halls and Dell (1966) and Martin *et al.* (1980).

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The basic concept behind RSS can be briefly described as follows. Suppose X_1, X_2, $\cdots$, X_n is a SRS from $F(x)$ with a mean μ and a finite variance σ^2. Then a standard nonparametric estimator of μ is $\overline{X}=\sum_1^n X_i/n$ with $\mathrm{var}(\overline{X})=\sigma^2/n$. In contrast to SRS, RSS uses only one observation, namely, $X_{1:n}\equiv X_{(11)}$, the lowest observation from this set, then $X_{2:n}\equiv X_{(22)}$, the second lowest from another independent set of n observations, and finally $X_{n:n}\equiv X_{(nn)}$, the largest observation from a set of n observations. This process can be described in a table as follows.

《Tab. 1.1》 Display of n^2 observations in n sets of n each

$X_{(11)}$	$X_{(12)}$	$\cdots$	$X_{(1(n-1))}$	$X_{(1n)}$
$X_{(21)}$	$X_{(22)}$	$\cdots$	$X_{(2(n-1))}$	$X_{(2n)}$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
$X_{(n1)}$	$X_{(n2)}$	$\cdots$	$X_{(n(n-1))}$	$X_{(nn)}$

McIntyre (1952) recommended measuring and using only $\{X_{(11)},\ldots,X_{(nn)}\}$, known as a RSS, for estimation of μ, and proposed the unbiased estimator

$$\hat{\mu}_{rss}=\sum_{i=1}^{n}X_{(ii)}/n \qquad \text{(1.1)}$$

as a rival estimator as opposed to $\overline{X}$. It was proved by Takahasi and Wakamoto (1968) that

$$\mathrm{var}(\hat{\mu}_{rss})<\mathrm{var}(\overline{X})\ !$$

Many aspect of RSS have been studied in the literature, and we refer to

Sinha *et al.* (1996) for a comprehensive account of this area of research.

Recently Sinha *et al.* (1996) , Lam *et al.* (1994) and Shen (1994a) investigated further improvements and suitable modifications of RSS for estimation of various parameters when the functional form of $F(x)$ is known, but its parameters are partially or completely unknown.

Assume that $F(x)$ is normal with an unknown mean μ and a known variance, and the problem is to test the hypothesis $H_0 : \mu = 0$ versus $H_1 : \mu > 0$. Based on RSS and its various modifications, as introduced in Sinha *et al.* (1996), Shen (1994b) and Shen and Yuan (1995) proposed a variety of exact tests for the above problem with much better power properties compared to the traditional normal test based on SRS.

In this paper we assume that $F(x)$ is normal with unknown mean μ and an unknown variance σ^2, and propose exact tests for both μ and σ^2 based on RSS. Towards this end, note that recently Yu *et al.* (1996) derived three unbiased estimators for σ^2 based on RSS :

(*i*)

$$\hat{\sigma}_1^2 = a_n \sum_1^n \frac{(X_{(ii)} - \hat{\mu}_{blue})^2}{\upsilon_i} \qquad \text{(1.2)}$$

where

$$a_n = \left(n - 1 + \sum_1^n \frac{\nu_i^2}{\upsilon_i} \right)^{-1} \qquad \text{(1.3)}$$

$$\hat{\mu}_{blue} = \frac{\sum_1^n X_{(ii)}/\upsilon_i}{\sum_1^n 1/\upsilon_i} \qquad \text{(1.4)}$$

and ν_i is the mean and υ_i is the variance of the ith order statistic in a sample of size n from a standard normal population.

(*ii*)

$$\hat{\sigma}_2^2 = g_n(b)\sum_1^n b_i(X_{(ii)} - \hat{\mu}_{blue})^2 \qquad \text{(1.5)}$$

where

$$g_n(b) = \left(\sum_1^n b_i\left(\nu_i^2 + \upsilon_i - \left(\sum_{j=1}^n \frac{1}{\upsilon_j}\right)^{-1}\right)\right)^{-1} \qquad \text{(1.6)}$$

where the optimal weights b_i's are determined such that the estimator has the smallest variance. Note that when $b_i = 1/\upsilon_i$ for all i, $\hat{\sigma}_2^2 = \hat{\sigma}_1^2$.

(*iii*)

$$\hat{\sigma}_3^2 = c_n\left(\sum_1^n \frac{\nu_i X_{(ii)}}{\upsilon_i}\right)^2 \qquad \text{(1.7)}$$

where

$$c_n = \left(\sum_1^n \frac{\nu_i^2}{\upsilon_i}\right)^{-1}\left(1 + \sum_1^n \frac{\nu_i^2}{\upsilon_i}\right)^{-1} \qquad \text{(1.8)}$$

Using the above RSS-based estimators of σ^2, we propose several tests of

$H_0:\mu=0$ versus $H_1:\mu>0$ and $H_0:\sigma^2=1$ versus $H_1:\sigma^2>1$. In Section **2**, tests for μ based on $\hat{\mu}_{rss}$, $\hat{\mu}_{blue}$ and $\hat{\sigma}_i^2, i=1,2,3$ are discussed. In Section **3**, tests for σ^2 based on $\hat{\sigma}_i^2, i=1,2,3$ are discussed. In Section **4**, we provide power comparisons of all these tests with the usual t test and chi-square test.

2. Tests for Normal Mean Based on RSS

In this section we propose a variety of tests for $H_0:\mu=0$ versus $H_1:\mu>0$ based on RSS and its modification.

Recall that the usual t test for H_0 based on a SRS of size n rejects H_0 if

$$\boldsymbol{t}=\frac{\sqrt{n}\bar{X}}{\sqrt{\sum_1^n(X_i-\bar{X})^2/(n-1)}}\geq t_{\alpha,n-1}$$

where $t_{\alpha,n-1}$ is the upper α level cut-off point of the Student t distribution with $n-1$ degrees of freedom, and its power at $\mu>0$ is given by

$$\boldsymbol{Power}\,(\mu\,|\,\boldsymbol{SRS})=\boldsymbol{P}\{\,t\geq t_{\alpha,n-1}\,|\,\mu\,\},$$

where t has a noncentral t distribution with $n-1$ degrees of freedom and noncentrality parameter $\sqrt{n}\mu/\sigma$.

2.1 Test based on McIntyre's $\hat{\mu}_{rss}$ and $\hat{\sigma}_i^2$

Since the statistic $\hat{\mu}_{rss}$ proposed by McIntyre(1952)in **(1.1)** is unbiased, we propose a test statistic based on $\hat{\mu}_{rss}$ and $\hat{\sigma}_i^2$ for H_0 and reject H_0 if

$$T_{rss,i} = \frac{\hat{\mu}_{rss}}{\hat{\sigma}_i} > c^i_{rss,\alpha}\,, \qquad i = 1, 2, 3\,,$$

with its power at $\mu > 0$ as

$$Power\,(\mu \mid T_{rss,i}) = P\,\{\, T_{rss,i} > c^i_{rss,\alpha} \mid \mu \,\}\,, \qquad i = 1, 2, 3\,,$$

where $c^i_{rss,\alpha}$ is the upper α level cut-off point of $T_{rss,i}$ under H_0. Clearly, under H_0, the statistic $T_{rss,i}$ is a pivot which implies that $c^i_{rss,\alpha}$ is an absolute constant depending only on n and α. We have provided in 《Tab. 2.1》 simulated values of these cut-off points along with their standard errors for $n = 2, \ldots, 10$ and $\alpha = 0.05$.

《Tab. 2.1》 Cut-off points $c^i_{rss,\alpha}$, $i = 1, 2, 3$, $\alpha = 0.05$

n	$c^1_{rss,\alpha}$		$c^2_{rss,\alpha}$		$c^3_{rss,\alpha}$	
	Mean	S.E.	Mean	S.E.	Mean	S.E.
2	3.19	0.01	3.19	0.01	3.19	0.01
3	1.03	0.002	1.01	0.002	1.48	0.005
4	0.67	0.001	0.66	0.001	0.81	0.003
5	0.50	0.0007	0.50	0.0007	0.56	0.001
6	0.41	0.0006	0.41	0.0006	0.47	0.0007
7	0.35	0.0005			0.36	0.0005
8	0.30	0.0004			0.31	0.0004
9	0.27	0.0004			0.28	0.0004
10	0.25	0.0003			0.25	0.0003

2.2 Test based on BLUE $\hat{\mu}_{blue}$ and $\hat{\sigma}_i^2$

Sinha *et al.* (1996) derived the BLUE of μ based on McIntyre's RSS $(X_{(11)},...,X_{(nn)})$, given in (1.4). We propose a test for H_0 based on $\hat{\mu}_{blue}$ and $\hat{\sigma}_i^2$, $i=1,2,3$, which rejects H_0 if

$$T_{blue,i}=\frac{\hat{\mu}_{blue}}{\hat{\sigma}_i}>c_{blue,\alpha}^i, \qquad i=1,2,3,$$

with its power at $\mu>0$ as

$$Power\,(\mu\,|\,T_{blue,i})=P\,\{\,T_{blue,i}>c_{blue,\alpha}^i\,|\,\mu\,\},$$

where $c_{blue,\alpha}^i$ is the upper α level cut-off point of $T_{blue,i}$ under H_0. Clearly, under H_0, the statistic $T_{blue,i}$ is again a pivot which implies that $c_{blue,\alpha}^i$ is an absolute constant depending only on n and α. We have provided in 《Tab. 2.2》 simulated values of these cut-off points along with their standard errors for $n=2,...,10$ and $\alpha=0.05$.

《Tab. 2.2》 Cut-off points $c_{blue,\alpha}^i$, $i=1,2,3$, $\alpha=0.05$

n	$c_{blue,\alpha}^1$		$c_{blue,\alpha}^2$		$c_{blue,\alpha}^3$	
	Mean	S.E.	Mean	S.E.	Mean	S.E.
2	3.19	0.01	3.19	0.01	3.19	0.01
3	1.03	0.002	1.01	0.002	1.48	0.005
4	0.66	0.001	0.66	0.001	0.81	0.003
5	0.50	0.0008	0.50	0.0007	0.55	0.001
6	0.41	0.0006	0.40	0.0006	0.43	0.0007
7	0.34	0.0005			0.36	0.0005
8	0.30	0.0004			0.31	0.0004
9	0.26	0.0004			0.27	0.0004
10	0.24	0.0003			0.24	0.0003

3. Tests for Normal Variance Based on RSS

In this section we propose a variety of tests for $H_0:\sigma^2=1$ versus $H_1:\sigma^2>1$ based on RSS.

Recall that the usual chi-square test for H_0 based on a SRS of size n rejects H_0 if

$$Q_{srs}=\sum_1^n (X_i-\overline{X})^2>\chi^2_{\alpha,n-1},$$

where $\chi^2_{\alpha,n-1}$ is the upper α level cut-off point of the Chi-square distribution with $n-1$ degrees of freedom.

We propose three test procedures for H_0 based on $\hat{\sigma}_1^2,\hat{\sigma}_2^2,\hat{\sigma}_3^2$ defined in (1.2), (1.5), (1.7) which reject H_0 if

$$Q_i=\hat{\sigma}_i^2>c_\alpha^i,\qquad i=1,2,3,$$

with power at $\sigma^2>1$ as

$$Power\,(\sigma^2\,|\,Q_i)=P\{\,Q_i>c_\alpha^i\,|\,\sigma^2\,\},\qquad i=1,2,3,$$

where c_α^i is the upper α level cut-off point of Q_i under H_0. Clearly, under H_0, the statistic Q_i is again a pivot which implies that c_α^i is an absolute constant depending only on n and α. We have provided in《Tab. 3.1》simulated values of these cut-off points along with their standard errors for $n=2,\ldots,10$ and $\alpha=0.05$.

《Tab. 3.1》 Cut-off points c_α^i, $i=1,2,3$, $\alpha=0.05$

n	c_α^1		c_α^2		c_α^3	
	Mean	S.E.	Mean	S.E.	Mean	S.E.
2	3.59	0.006	3.59	0.006	3.59	0.006
3	2.71	0.004	2.65	0.003	3.03	0.004
4	2.30	0.003	2.29	0.003	2.59	0.003
5	2.06	0.002	2.04	0.002	2.29	0.002
6	1.90	0.002	1.89	0.002	2.08	0.002
7	1.78	0.001			1.93	0.002.
8	1.69	0.001			1.81	0.001
9	1.62	0.001			1.72	0.001
10	1.57	0.0009			1.65	0.001

4. Comparison of Powers of SRS and RSS Schemes

In this section we provide a power comparison for all the tests discussed in Section **2** and Section **3** with the usual t test and chi-square test, respectively. The simulation program was written in SAS running on PC586. For values of v_i and υ_i, we have used the tables from Arnold *et al.* (1992) and Tietjen *et al.* (1997).

In each case of $n=2,...,10$, μ=0, 0.25, 0.75, 1.0, and σ^2=0, 0.25, 0.5, 0.75, 1.0, the simulation contains 1,000 samples, each sample being replicated 1,000 times, and we have taken $\alpha=0.05$. Since the values of b_i in **(1.5)** are available only for $n=2,...,6$ in Yu *et al.* (1996), we took $n=2,...,6$ for every test statistic related to $\hat{\sigma}_2^2$.

《Tab. 4.1》shows, for every n and μ, simulated powers of the tests based on t, $T_{rss,i}$ along with their standard errors. 《Tab. 4.2》shows the same for $T_{blue,i}$. As expected, tests based on $\hat{\mu}_{rss}$ and $\hat{\mu}_{blue}$ perform much better than the traditional t test. 《Tab. 4.3》shows, for every n and σ^2, simulated powers of the tests based on $\hat{\sigma}_i^2$, $i=1,2,3$ along with their standard errors. As expected, test based on each $\hat{\sigma}_i^2$ performs much better than the traditional chi-square test.

Remark 1. Based on the simulated powers given in the above tables, we can recommend the use of $T_{rss,1}$ and $T_{blue,1}$ for μ, and Q_1 (and Q_2, if available) for σ^2.

Remark 2. The usual t and chi-square tests have the advantages of with exact cut-off points. However, the test procedures based on RSS scheme use the approximated ones.

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《Tab. 4.1》 Powers of tests t, $T_{rss,i}$, $i=1,2,3$

n	μ	t	$T_{rss,1}$		$T_{rss,2}$		$T_{rss,3}$	
			Mean	S.E.	Mean	S.E.	Mean	S.E.
2	0.25	0.076	0.079	0.003	0.079	0.003	0.079	0.003
	0.50	0.103	0.123	0.003	0.123	0.003	0.123	0.003
	0.75	0.142	0.178	0.004	0.178	0.004	0.178	0.004
	1.00	0.180	0.228	0.004	0.228	0.004	0.228	0.004
3	0.25	0.093	0.124	0.003	0.123	0.003	0.108	0.003
	0.50	0.152	0.238	0.004	0.243	0.004	0.178	0.004
	0.75	0.232	0.403	0.005	0.409	0.005	0.286	0.005
	1.00	0.327	0.567	0.005	0.576	0.005	0.399	0.005
4	0.25	0.100	0.174	0.004	0.175	0.004	0.152	0.004
	0.50	0.185	0.382	0.005	0.382	0.005	0.311	0.005
	0.75	0.302	0.633	0.005	0.635	0.005	0.525	0.005
	1.00	0.444	0.829	0.004	0.830	0.004	0.719	0.004
5	0.25	0.121	0.215	0.004	0.216	0.004	0.194	0.004
	0.50	0.231	0.526	0.005	0.531	0.005	0.477	0.005
	0.75	0.398	0.808	0.004	0.810	0.004	0.757	0.004
	1.00	0.578	0.949	0.002	0.950	0.002	0.921	0.003
6	0.25	0.135	0.271	0.004	0.271	0.004	0.257	0.004
	0.50	0.280	0.665	0.005	0.663	0.005	0.632	0.005
	0.75	0.478	0.913	0.003	0.915	0.003	0.891	0.003
	1.00	0.6806	0.990	0.001	0.9911	0.0009	0.984	0.001
7	0.25	0.146	0.332	0.005			0.322	0.005
	0.50	0.329	0.775	0.004			0.753	0.004
	0.75	0.557	0.970	0.002			0.964	0.002
	1.00	0.7641	0.9977	0.0005			0.9971	0.0005
8	0.25	0.161	0.400	0.005			0.390	0.005
	0.50	0.362	0.858	0.003			0.846	0.004
	0.75	0.6101	0.9916	0.0009			0.989	0.001
	1.00	0.8194	0.9998	0.0001			0.9996	0.0002
9	0.25	0.169	0.466	0.005			0.456	0.005
	0.50	0.399	0.918	0.003			0.910	0.003
	0.75	0.6597	0.9975	0.0005			0.9974	0.0005
	1.00	0.8645	1	0			0.9999	0.0001
10	0.25	0.182	0.532	0.005			0.524	0.005
	0.50	0.427	0.958	0.002			0.954	0.002
	0.75	0.7094	0.9994	0.0002			0.9993	0.0003
	1.00	0.8984	1	0			1	0

《Tab. 4.2》 Powers of tests t, $T_{blue,i}$, $i=1,2,3$

n	μ	t	$T_{blue,1}$		$T_{blue,2}$		$T_{blue,3}$	
			Mean	S.E.	Mean	S.E.	Mean	S.E.
2	0.25	0.076	0.079	0.003	0.079	0.003	0.079	0.003
	0.50	0.103	0.123	0.003	0.123	0.003	0.123	0.003
	0.75	0.142	0.178	0.004	0.178	0.004	0.178	0.004
	1.00	0.180	0.228	0.004	0.228	0.004	0.228	0.004
3	0.25	0.093	0.123	0.003	0.122	0.003	0.105	0.003
	0.50	0.152	0.236	0.004	0.240	0.004	0.178	0.004
	0.75	0.232	0.400	0.005	0.408	0.005	0.284	0.005
	1.00	0.327	0.565	0.005	0.577	0.005	0.398	0.005
4	0.25	0.100	0.176	0.004	0.176	0.004	0.151	0.004
	0.50	0.185	0.380	0.005	0.381	0.005	0.307	0.005
	0.75	0.302	0.636	0.005	0.638	0.005	0.522	0.005
	1.00	0.444	0.831	0.004	0.833	0.004	0.720	0.004
5	0.25	0.121	0.217	0.004	0.217	0.004	0.193	0.004
	0.50	0.231	0.535	0.005	0.535	0.005	0.483	0.005
	0.75	0.398	0.815	0.004	0.817	0.004	0.763	0.004
	1.00	0.578	0.954	0.002	0.954	0.002	0.925	0.003
6	0.25	0.135	0.272	0.004	0.273	0.004	0.257	0.004
	0.50	0.280	0.676	0.005	0.675	0.005	0.639	0.005
	0.75	0.478	0.923	0.003	0.924	0.003	0.902	0.003
	1.00	0.6806	0.9927	0.0009	0.9930	0.0008	0.988	0.001
7	0.25	0.146	0.340	0.005			0.327	0.005
	0.50	0.329	0.791	0.004			0.772	0.004
	0.75	0.557	0.977	0.001			0.970	0.002
	1.00	0.7641	0.9990	0.0003			0.9985	0.0004
8	0.25	0.161	0.413	0.005			0.401	0.005
	0.50	0.362	0.879	0.003			0.865	0.003
	0.75	0.6101	0.9942	0.00085			0.9927	0.0009
	1.00	0.8194	1	0			1	0
9	0.25	0.169	0.489	0.005			0.482	0.005
	0.50	0.399	0.935	0.002			0.927	0.003
	0.75	0.6597	0.9992	0.0003			0.9991	0.0003
	1.00	0.8645	1	0			1	0
10	0.25	0.182	0.558	0.005			0.550	0.005
	0.50	0.427	0.970	0.002			0.968	0.002
	0.75	0.7094	0.9999	0.0001			0.9999	0.0001
	1.00	0.8984	1	0			1	0

《Tab. 4.3》 Powers of tests Q_{srs}, Q_1, Q_2 and Q_3

n	σ^2	Q_{srs}	Q_1		Q_2		Q_3	
			Mean	S.E.	Mean	S.E.	Mean	S.E.
2	1.25	0.080	0.084	0.003	0.084	0.003	0.084	0.003
	1.50	0.110	0.122	0.003	0.122	0.003	0.122	0.003
	1.75	0.139	0.154	0.004	0.154	0.004	0.154	0.004
	2.00	0.166	0.185	0.004	0.185	0.004	0.185	0.004
3	1.25	0.091	0.096	0.003	0.098	0.003	0.093	0.003
	1.50	0.136	0.158	0.004	0.161	0.004	0.147	0.004
	1.75	0.181	0.204	0.004	0.208	0.004	0.185	0.004
	2.00	0.224	0.260	0.004	0.266	0.004	0.229	0.004
4	1.25	0.100	0.106	0.003	0.107	0.003	0.097	0.003
	1.50	0.157	0.186	0.004	0.188	0.004	0.165	0.004
	1.75	0.215	0.268	0.004	0.271	0.004	0.230	0.004
	2.00	0.272	0.331	0.005	0.335	0.005	0.281	0.004
5	1.25	0.108	0.125	0.003	0.126	0.003	0.112	0.003
	1.50	0.176	0.221	0.004	0.226	0.004	0.189	0.004
	1.75	0.247	0.317	0.005	0.326	0.005	0.272	0.005
	2.00	0.315	0.406	0.005	0.414	0.005	0.337	0.005
6	1.25	0.115	0.141	0.003	0.144	0.004	0.129	0.003
	1.50	0.194	0.248	0.004	0.250	0.004	0.216	0.004
	1.75	0.276	0.380	0.005	0.382	0.005	0.322	0.005.0
	2.00	0.354	0.477	0.005	0.484	0.005	0.400	0.005
7	1.25	0.122	0.149	0.004			0.137	0.003
	1.50	0.211	0.290	0.005			0.251	0.004
	1.75	0.303	0.430	0.005			0.363	0.005
	2.00	0.391	0.555	0.005			0.472	0.005
8	1.25	0.128	0.170	0.004			0.155	0.004
	1.50	0.227	0.327	0.005			0.276	0.004
	1.75	0.329	0.480	0.005			0.408	0.005
	2.00	0.425	0.624	0.005			0.539	0.005
9	1.25	0.134	0.179	0.004			0.159	0.004
	1.50	0.242	0.360	0.005			0.319	0.005
	1.75	0.354	0.534	0.005			0.461	0.005
	2.00	0.458	0.681	0.005			0.600	0.005
10	1.25	0.140	0.200	0.004			0.179	0.004
	1.50	0.257	0.397	0.005			0.350	0.005
	1.75	0.378	0.593	0.005			0.523	0.005
	2.00	0.489	0.736	0.004			0.658	0.005

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利用排序集合樣本對常態母體參數作檢定

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摘要

某些情況下，研究之實驗或抽樣樣本之排序（未經過儀器測量）要比測度容易得到，基於這個原因，McIntyre（1952）提出了"排序集合抽樣"（ranked set sample）的概念。利用此方法去估計母體平均數所得到的不偏估計式，和我們通常所用的"簡單隨機抽樣"的不偏估計式來作比較，其變異數要來得小。

在本研究中，我們將此概念推廣至對常態母體的參數作檢定。利用 Sinha 等人（1996）及 Yu 等人（1996）所建議的一些平均數與變異數的估計式去構造一些新的檢定式，並和常用的簡單隨機抽樣對平均數與變異數作檢定的 t 檢定和卡方檢定作比較。經過模擬的結果，在"量測"樣本數相同的情況下，我們所建議的檢定式的檢定力皆要比 t 檢定和卡方檢定的檢定力來得大。

關鍵詞：順序統計量、檢定力、排序集合抽樣、簡單隨機抽樣。

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