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Strategic Product Life Cycle Forecasting System

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Abstract

Stage concerns have been important in product life cycles. Such concerns are stage identification, stage-related strategies and, here newly introduced, 'stage modeling'. Stage modeling is concerned with not only modeling but also aggregation of individual stages in an overall-influencing manner. It not only preserves the respective characteristics of the stages but also may be explored for the stage-related strategy issue later. To date, this aspect of PLC modeling has not yet been explored. In this paper, a fuzzy PLC modeling capable of preserving the fuzzy individual characteristics of the stages is proposed. The various concepts such as boundary identification, fuzzy stages modeling, and stages' inter-influence functions are discussed. Finally, to illustrate the approach, a numerical example is provided.

Keywords: Product life cycle; Forecasting; Multistage fuzzy regression; Stage modeling; Stage identification

一、Introduction

Over the past four decades the product life cycle (PLC) concept has been widely discussed and utilized for a number of purposes such as product management, strategic planning, cost and financial aspects, retailing, purchasing, international trade and production planning and inventory control (e.g. see [20]) by a number of researches (e.g. see [10, 13]). The observed patterns of PLCs also indicate the significant role of the 'stage' concerns. Such concerns represent three

interrelated aspects among those in the PLC concept:

- Stage (boundary) identification (e.g. boundaries between introduction, growth, maturity, and decline stages)
- Stage-related strategic actions and the impacts relating to the marketing and business planning
- Stage modeling for an overall PLC modeling (to be introduced here)

二、Fuzzy numbers and fuzzy regression analysis

In this section, fuzzy numbers are defined and the fuzzy regression analysis (FRA) is reviewed.

The fuzzy numbers will be represented by the L-R type representation [9]: $M = (m, \alpha, \beta)LR$, where m represents the mode, α and β the left and right spreads, respectively, and L and R the left and right reference (shape) functions, respectively, of M . This representation can be termed the spread form. A second equivalent form can also be used, i.e. the bound form: $M = (l, m, u)_{L-R}$, where $l (= m - \alpha)$ and $u (= m + \beta)$ denote respectively the lower and upper bounds of M . For fuzzy arithmetic operations, the approximate formulae (e.g., see [9, 4]) can be used, due to the fuzzy or approximate nature of most of the problems.

Let a general fuzzy linear model (GFLM) be defined as:

$$Y = f(\mathbf{x}, \mathbf{A}) = A_1 x_1 + \dots + A_w x_w = \mathbf{A}^T \mathbf{x} \quad (1)$$

$$\sim_{A_j}(a_j) = \begin{cases} 1 - |m_j - a_j|/c_j, & \text{if } l_j \leq a_j \leq u_j, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

$$\begin{aligned}
Y &= (l_1, m_1, u_1)_L x_1 + \cdots + (l_w, m_w, u_w)_L x_w \\
&= \left(\sum_{j=1}^w l_j x_j, \sum_{j=1}^w m_j x_j, \sum_{j=1}^w u_j x_j \right)_L,
\end{aligned} \tag{3}$$

$$\tilde{r}_Y(y) = \begin{cases} 1 - |y - m^T x| / c^T x, & \text{if } |y - m^T x| \leq c^T x \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

$$\text{Min } J = \sum_{p=1}^N c^T x_p, \tag{5}$$

三、Fuzzy stage characteristic-preserving PLC (FSCP-PLC) Modeling

As stated earlier, stage characteristic-preserving modeling depending on the nature of the problems requires a number of stages to be identified and modeled. A stage model can thus represent the characteristic of the stage. For instance, a PLC may contain the introduction (I), growth (G), maturity (M), and decline (D) stages. A further stage may be considered that divides the decline stage into two stages: decline and expiration (E) stages. Then, the expiration stage refers to the long right tail in the PLC.

$$S(t) = \sum_{i=1}^n S_i(t) \times W_i(t), \tag{6}$$

$$\sum_{i=1}^n W_i(t) = 1, \quad \forall t. \tag{7}$$

$$\begin{aligned}
\text{Min } & \sum_{t=1}^N \left(s(t) - \sum_{i=1}^n S_i(t) \times W_i(t) \right)^2 \\
\text{subject to } & \sum_i W_i(t) = 1, \quad \forall t.
\end{aligned} \tag{8}$$

3.1. Identification of stages in PLC

Identification of the stages (boundaries) in a PLC may precede PLC modeling. The following presents an assisting method that combines moving average and simple linear regression. The purpose of this method is to aid in identifying the boundaries (or transitional points) of the stages.

Algorithm 1.

(Short-term (p-period) moving regression)

Step 1. Determine the number of periods p for the ST-MR. Set k = p.

Step 2. Run a simple (non-fuzzy) linear regression: $\hat{s}(t) = a + bt$ on data (s(t), t), t = (k-p+1), ..., k, and let the result be b(k) = b.

Step 3. If k is smaller than N, increase k by one and return to step 2. Otherwise, terminate, and plot the results (b(k), k), k = p, ..., N.

Algorithm 2.

(Boundary identification)

Step 1. For the boundary between stages I and G, first refer to the (s(t), t)-plot and approximately determine a candidate point or region.

Step 2. Zoom into the area of the candidate point or region corresponding in the (b(k), k)-plot. Confirm the transitional point(s) (denoted as kI|G) by that between point k that immediately follows kI|G and kI|G, b(k) shows a significant increase in the region. If no decisive action can be taken, the transitional point may be set to the suggestion listed in Table 1 as given by the other researchers.

Step 3. Then, for stages G and M, repeat steps 1 and 2, but confirm the transitional point(s) (as kG|M) by that between k immediately following kG|M and kG|M in the region, (1) b(k) increase shows a significant slowdown, (2) b(k) remains about a given level, or (3) b(k) suddenly decreases.

Step 4. Then, for stages M and D, repeat steps 1 and 2, but confirm the transitional point(s) (as kM|D) by that between k immediately preceding kM|D and kM|D, b(k) shows a sudden or significant decrease in the region.

Step 5. For stages D and E, repeat Steps 1 and 2, but confirm the transitional point(s) (as kD|E) by that between k immediately preceding kD|E and kD|E in the region, (1) b(k) decrease shows a significant slowdown, (2) b(k) remains about a given level, or (3) b(k) significantly increases.

3.2. Construction of the stage models

As a variety of functional forms have been suggested in the literature, here for a demonstration the GFLM may be illustrated

for the stage models. A stage model may be defined as:

$$S_i(t) = f_i(\tau, A_i) \\ = A_{i,0} + A_{i,1}t + \dots + A_{i,w}t^w = (A_i)^T \mathbf{t} \quad (9)$$

$$\text{Min} \quad J_i = \sum_{t \in X_i} (c_i)^T \mathbf{t}, \quad (10)$$

The linear program (10) is solved for each stage on the stage's relevant data. Also, with the fuzzy stage characteristic-preserving capability, the PLC modeling can be readily improved to modeling more flexible PLCs, as can be seen in the later example.

$$c_i = \begin{cases} (1 + k_{i+1,U})/2, & \text{for } i = 1, \\ (k_{i-1,L} + k_{i+1,U})/2, & \text{for } i = 2, \dots, (n-1), \\ (k_{n-1,L} + N)/2, & \text{for } i = n, \end{cases} \quad (11)$$

3.3. Stage inter-influence functions

As stated earlier, the stage inter-influence functions are to make the stages capable of appropriately interacting with other stages when combined together into an overall PLC.

Definition 1.

A stage's characteristic (forecasts) should be constructed and mainly affected by the stage and yet, affected by the other stages too. $W_i(t)$, $i = 1, \dots, n$, can be defined as:

$$W_i(t) = \begin{cases} y_1, & \text{for } i = 1 \text{ and } t \leq c_1, \\ g_i(t), & \text{for } i = 2, \dots, (n-1), i = 1 \text{ and } t > c_1, \\ y_n, & \text{for } i = n \text{ and } t > c_n, \end{cases} \quad (12)$$

Algorithm 3.

(The FSCP-PLC modeling)

Step 1. Identify stages of the PLC by using the short-term moving regression, Algorithm 1 and boundary identification, Algorithm 2. Then, calculate the mid-stages, Eq. (11).

Step 2. Determine the functional form(s) of stages $S_i(t)$.

Step 3. Construct the stage models $S_i(t)$ by the fuzzy regression analyses, Eq. (10).

Step 4. Identify the inter-influence functions of these stages, Definition 1 or 1-1.

Step 5. Construct the overall PLC model, Eq. (6). Plot the results ($S(t)$, t), $t = 1, \dots, N$.

Step 6. If the results are satisfactory, terminate; otherwise, for a suspicious stage(s), the following action may be taken: (1) revise

the transitional point(s) by either moving the transitional point to the right or left or by enlarging or reducing it and return to step 3, (2) revise the functional form(s) of $S_i(t)$ and return to step 3, or (3) revise transitional weights $'_{i+1}$, $''_{i+1}$ or function(s) $g_i(t)$ (or simply shape parameters X'_{i+1} , X''_{i+1}) and return to step 5.

四、Numerical example

To demonstrate how the proposed approach can work for PLC modeling, in this section we show an example that consists of PLC time-series data similar to Cox's finding as mentioned in Section 3 (Fig. 2 and Table 2). Five stages are to be identified and are considered as appropriate. They are I, G, M, D stages and an additional stage that may be termed as 'Reflex or reverberation stage' post stage Decline, or stages $i = 1, \dots, 5$. Meanwhile, in Fig. 2, the result of a conventional (non-fuzzy) polynomial regression analysis on the entire data set ($t = 1, \dots, 55$) is also provided with $\hat{s}(t) = 153280.08 - 72418.32t + 7974.06 t^2 - 238.57 t^3 + 2.15 t^4$, which however shows far from a best fit

五、Conclusion

In this paper, we have proposed a fuzzy PLC modeling capable of preserving the fuzzy individual characteristics of the stages in the PLC. The approach has utilized the concepts as short-term moving regression, fuzzy stage modeling, and inter-influence functions for the various aspects such as boundary identification, stage modeling, and aggregation of individual stages into the overall PLC results. The advantages with this approach are its flexibility and adaptability. As may be observed, other functional forms of the PLCs can be adapted in this approach, too.

Future research that stems from this research could be in many directions. For example, the well described Bass new-product-diffusion model and its extended forms [1, 13] such as that extended for individual adoption process, time-varying parameter, market potential, marketing

strategies, product and market characteristics, and supply restriction may be carried out with the present approach as to explore better modeling and adaptation of the PLC concepts

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