行政院國家科學委員會補助專題研究計畫 ☑期中進度報告

低維量子自旋系統之研究

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計畫編號：NSC 96-2112-M-029 -004 -MY3
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計畫主持人：楊明峰

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行政院國家科學委員會專題研究計畫成果報告

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Study on low-dimensional quantum spin systems

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一、中文摘要

在本計畫中，我們利用張量重整化群方法(tensor-renormalization group approach)來研究在Shastry-Sutherland晶格上的古典反鐵磁Ising 模型的磁化行為。這個方法可以輕易的處理大尺寸系統，因而避免有限尺寸效應所造成的影響。我們發現，此系統只會有一個具有1/3飽和磁化的磁化平台。我們也計算了不同的磁耦合常數與溫度對這個磁化平台所造成的影響。

關鍵詞：Ising 模型，Shastry-Sutherland 晶格，張量重整化群，磁化平台

Abstract

We study the magnetization for the classical antiferromagnetic Ising model on the Shastry-Sutherland lattice using the tensor renormalization-group approach. With this method, one can probe large spin systems with little finite-size effect. For a range of temperature and coupling constant, a single magnetization plateau at one third of the saturation value is found. We investigate the dependence of the plateau width on temperature and on the strength of magnetic frustration. Furthermore, the spin configuration of the plateau state at zero temperature is determined.

Keywords: Ising model, Shastry-Sutherland lattice, tensor renormalization group, magnetization plateau

二、緣由與目的、結果與討論

The frustrated spin systems have attracted much attention over last decades since very rich physics can appear in these systems [1]. Some interest in such systems is concentrated on fascinating sequence of magnetization plateaus at fractional values of the saturation magnetization, which was first observed in two-dimensional spin-gap material $SrCu_2(BO_3)_2$ [2]. This compound can be described well by spin-1/2 antiferromagnetic Heisenberg model on the frustrated Shastry-Sutherland lattice (or the orthogonal-dimer lattice) [3]. Besides the previously discovered plateaus at 1/3, 1/4 and 1/8 of the saturated magnetization, evidence in favor of more fractional magnetization plateaus down to values as small as 1/9 has been reported recently [4-6]. Stimulated by the discovery of magnetization plateaus, various theoretical and experimental explorations have been devoted to the properties of the Shastry-Sutherland model in a magnetic field [7-9].

Similar phenomena of magnetization plateaus is also observed in rare-earth tetraborides RB_4. The magnetic ions of these compounds are again located on a lattice that is topologically equivalent to the Shastry-Sutherland lattice [10-16]. In particular, magnetization plateaus at small fractional values (1/7, 1/9, … of the saturation magnetization) are reported in the compound TmB_4 [15-16]. Because fully polarized state can be reached for experimentally accessible magnetic fields, this compound allows exploration of its complete magnetization process. Note that, due to large total magnetic moments of the magnetic ions, this compound can be considered as a classical system. Moreover, because of strong crystal field effects, the effective spin model for TmB_4 has been suggested to be described by the spin-1/2 Shastry-Sutherland model under strong Ising (or easy-axis) anisotropy [16]. Thus, studying the Ising limit is the first step toward a complete understanding of the magnetization process for this material.

In order to check theoretically if other reported magnetization plateaus at small fractional values can be stabilized in the current model, unbiased large-scale calculations are called for. This is because the unit cells of magnetization profiles inside high-commensurability plateaus are usually quite large, calculations for systems of finite sizes may prevent reliable predictions for these cases. Therefore, to avoid the frustration for certain magnetization plateaus coming from geometric constraints, and in particular to uncover the possibility of plateaus at small fractional values, analyzing systems of large enough sizes are necessary.

Lately, based on ideas from quantum information theory, the tensor renormalization group (TRG) method is developed [17], which can efficiently calculate quantities of classical systems of very large sizes. This technique can in principle be applied to any classical lattice with local interactions as long as the partition function can be expressed as a tensor network [18]. Because the accuracy can be systematically improved by increasing the cutoff on the index range of the tensors, highly precise quantities can be calculated under the TRG approach even in the thermodynamic limit [17,19,20]. Therefore, the TRG method is one of the most suitable ways to study the magnetization process of the classical frustrated spin systems in the thermodynamical limit.

In the present work, the magnetization process of the spin-1/2 Shastry-Sutherland model in the Ising limit is investigated by employing the TRG approach [17,19,20]. We find that the magnetization curve exhibits exactly one plateau at 1/3 of the saturation value. Our results are in accordance with the findings in Ref. [21]. Since there is no evidence for the presence of any additional plateaus for the spin-1/2 Shastry-Sutherland model in the Ising limit, to explain the experimental results, one must go beyond this simple model.

三、計畫成果自評

由上述的結果可以看出，我們的工作一方面釐清了 Shastry-Sutherland 晶格上的古典反鐵磁 Ising 模型被用來解釋 TmB_4 的磁化行為的適用性；另一方面亦提供了張量重整化群方法在古典統計力學的應用的一個新例證。這對於國內外相關的後續研究工作有著相當程度的幫助。

此外，相關的研究成果[22]均已刊登至 Physical Review B。

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