

# 行政院國家科學委員會專題研究計畫 成果報告

## 二階段抽樣法在非對稱損失函數下貝氏序列估計之研究 研究成果報告(精簡版)

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中文摘要： 貝氏的架構下，考慮研究使用非對稱的 LINEX(linear exponential)損失函數來估計特殊一維指數族(one-parameter exponential family)分佈的平均值並且每個觀察值有一固定成本的序列估計問題。本研究計畫對 LINEX 損失函數的貝氏序列估計問題，在給定事先分佈(prior distribution)下，提出二階段法則(two-stage procedure)並證明它具有漸近點最優(asymptotically pointwise optimal)和漸近最佳(asymptotically optimal)性質。除此之外，將提出一個具有穩健性(robust)的二階段法則，此法則與資料的分佈、事先分佈無關，並將證明在某些條件下的事先分佈，它如同給定事先分佈下的漸近點最優法則所具有的漸近性質。

中文關鍵詞： 漸近最佳性，漸近點最優，LINEX 損失函數，序列估計，二階段法則。

英文摘要：

英文關鍵詞：

# A Robust Two-Stage Procedure in Bayes Sequential Estimation of a Particular Exponential Family Under LINEX Loss

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## Abstract

The problem of Bayes sequential estimation of the unknown parameter in a particular exponential family of distributions is considered under LINEX loss function for estimation error and a fixed cost for each observation. Instead of fully sequential sampling, a two-stage sampling technique is introduced to solve the problem in this paper. The proposed two-stage procedure is robust in the sense that it does not depend on the parameters of the conjugate prior. It is shown that the two-stage procedure is asymptotically pointwise optimal and asymptotically optimal for a large class of the conjugate priors.

**Keywords:** Asymptotically optimal; Asymptotically pointwise optimal; Bayes sequential estimation; LINEX loss function; Two-stage procedure.

Mathematical Subject Classifications: 62L12.

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## 1. Introduction

The Bayes sequential estimation problem is to seek an optimal sequential procedure which includes an optimal stopping time and a Bayes estimate. The Bayes estimate is usually obtained in the problem. Hence the Bayes sequential estimation problem is reduced to finding an optimal stopping time.

It is well known that the optimal stopping time exists in Bayes sequential estimation problem in certain case under mild regularity conditions. However, the exact determination of optimal stopping time appears to be a formidable task, in practice; see, e.g., Chow, Robbins and Siegmund (1971). Due to this difficulty in finding explicit optimal rules, some procedures have been proposed with the goal of finding "asymptotically" optimal rules. For instance, Bickel and Yahav (1967) provided a simple but very attractive large sample approximation to optimal rules, namely "asymptotically pointwise optimal" (APO) rules. Since Bickel and Yahav's initiation of this idea, many papers have envolved on developing APO rules in various different contexts; see, e.g. Bickel and Yahav (1968), Gleser and Kunte (1976), Woodroffe (1981), Martinsek (1987), Ghosh and Hoekstra (1995), Hwang (2001) and Hwang and Karunamuni (2008), among others.

From the practical standpoint, purely sequential procedures suffer, especially when money and time are important design factors. Multistage methods of sampling techniques are used in statistical inference. The idea of group sampling done in two stages from a normal population is proposed by Stein (1945). Cox (1952) extended the double sampling techniques to cover a wider class of problems. Within the classical non-Bayesian framework, Mukhopadhyay (1980) and Ghosh and Mukhopadhyay (1981) proposed two-stage procedures instead of purely sequential procedures in sequential interval and point estimation problems. The two-stage sampling techniques are applied to Bayes sequential estimation for the exponential distribution under the squared error loss by Hwang (1999).

In this paper, the problem of Bayes sequential estimation of the unknown parameter in a particular exponential family of distributions with LINEX loss function and fixed cost for each observation is considered. Given a conjugate prior, Jokiel-Rokita (2011) derived an APO procedure depending on the parameters of the prior distribution, and it is shown to be asymptotically optimal (AO). Instead of fully sequential sampling, a two-

stage sampling technique is introduced to solve the problem in Section 2. The proposed two-stage procedure is robust in the sense that it does not depend on the parameters of the prior distribution. It is shown that the two-stage procedure also shares the asymptotic properties with the APO procedure. The proofs of some auxiliary lemmas in order to obtain main theorems in Section 2 are given in Section 3.

## 2. Two-stage procedure and its asymptotic properties

Let  $X_1, X_2, \dots$  be a sequence of independent and identically distributed random variables from a particular exponential family of distributions with a density function of the form

$$f_{\theta}(x) = \theta^{\frac{k}{2}} e^{-s(x)\theta}, \quad x \in R,$$

with respect to some  $\sigma$ -finite measure, where  $\theta > 0$  is an unknown parameter,  $k$  is a positive constant, and  $s(x)$  is a nonnegative function. The particular exponential family of distributions was introduced by Rahman and Gupta (1993). In the case  $k$  is a positive integer, the family is called transformed chi-square family. Some distributions belong to the particular exponential family, for example, the normal distribution with known mean, the gamma distribution with known shape parameter and the Pareto distribution with known scale parameter. More details can be referred to Table 1 of Jokiel-Rokita (2011).

Suppose that we are interested in estimating  $\theta$ . Having recorded  $n$  observations  $X_1, \dots, X_n$ , we assume that the loss incurred in estimating  $\theta$  by  $d_n = d_n(X_1, \dots, X_n)$  is  $L(\theta, d_n) + cn$ , where

$$L(\theta, d_n) = \exp(a(d_n - \theta)) - a(d_n - \theta) - 1, \quad a \neq 0,$$

is the LINEX loss and  $c > 0$  is the cost for each observation. One notes that  $bL(\theta, d_n)$  with  $b > 0$  is a general form of the LINEX loss function, hence  $c$  can also be regarded as a relative weight with respect to the general form of the LINEX loss. The LINEX loss function was first introduced by Varian (1975). It is a very useful asymmetric loss function that increases approximately exponentially on one side of zero and approximately linearly on the other side. In the case  $a < 0$ , the loss function indicates that underestimation is more costly than overestimation. The opposite is true when  $a > 0$ .

Suppose that  $\theta$  has a gamma prior distribution  $\Gamma(\alpha, \lambda)$  with a density function of the

form

$$\pi(\theta) = \frac{\lambda^\alpha \theta^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda\theta}, \quad \theta > 0,$$

where  $\alpha > 0$  and  $\lambda > 0$ . For convenience, we denote  $\mathcal{F}_n = \sigma(X_1, \dots, X_n)$ ,  $S_n = \sum_{i=1}^n s(X_i)$  and  $\bar{S}_n = \frac{S_n}{n}$  for all  $n \geq 1$ . One notes that for given  $\theta$ ,  $s(X_i)$  has a gamma distribution  $\Gamma(\frac{k}{2}, \theta)$  and then  $\bar{S}_n$  has a gamma distribution  $\Gamma(\frac{nk}{2}, n\theta)$ . It is easy to see that the posterior distribution of  $\theta$  given  $\mathcal{F}_n$  is the gamma distribution  $\Gamma(\alpha_n, \lambda_n)$ , where  $\alpha_n = \alpha + \frac{nk}{2}$  and  $\lambda_n = \lambda + S_n$ . By straightforward calculations and assume  $a > -\lambda$ , we can obtain that for a given stopping time  $T$ , the optimal estimate is the Bayes estimate

$$\tilde{\theta}_T = \frac{\alpha_T}{a} \log \left( 1 + \frac{a}{\lambda_T} \right).$$

Then the Bayes risk of the Bayes sequential procedure  $(T, \tilde{\theta}_T)$  is equal to

$$E \left\{ \frac{a\alpha_T}{\lambda_T} - \alpha_T \log \left( 1 + \frac{a}{\lambda_T} \right) + cT \right\}.$$

Hence, finding an optimal Bayes sequential procedure for the sequential problem is equivalent to constructing an optimal stopping time for the sequence  $\{L_n(c); n \geq 1\}$ , where  $L_n(c) = Y_n + cn$  and

$$Y_n = \frac{a\alpha_n}{\lambda_n} - \alpha_n \log \left( 1 + \frac{a}{\lambda_n} \right).$$

Bickel and Yahav (1967, 1968) described methods for finding a family of stopping times  $\{t_c; c > 0\}$  which is APO with respect to  $\{Y_n + cn; n \geq 1\}$ , that is,

$$\lim_{c \rightarrow 0} \frac{Y_{t_c} + ct_c}{\inf_n E(Y_n + cn)} = 1 \quad \text{a.s.}$$

They also showed that this family of stopping times is AO, that is,

$$\lim_{c \rightarrow 0} \frac{E(Y_{t_c} + ct_c)}{\inf_T E(Y_T + cT)} = 1,$$

where the infimum extends over all  $\mathcal{F}_n$ -stopping times  $T$ .

To this case, Jokiel-Rokita (2011) showed that the sequence  $\{L_n(c); n \geq 1\}$  satisfies the conditions of Theorem 2.1 in Bickel and Yahav (1967) and Theorem 3.1 in Bickel and Yahav (1968). Hence Jokiel-Rokita (2011) obtained that under the condition  $a > -\lambda$ , the family of stopping times  $\{N_c; c > 0\}$  defined by

$$N_c = \inf \left\{ n \geq \left\lceil \frac{4}{k} \right\rceil + 1 : Y_n \leq cn \right\}, \quad c > 0,$$

with  $[x]$  denoting the integer part of  $x$ , is APO and AO with respect to the sequence  $\{L_n(c); n \geq 1\}$ .

The sequential procedure  $(N_c, \hat{\theta}_{N_c})$  depends on the known parameters of the gamma prior. When the parameters of the prior distribution are misspecified or unknown, the procedure is not appropriate. Hence, we would like to propose a procedure, which is independent of prior parameters, but at the same time it still possesses some asymptotic properties. Here we concentrate on a two-stage procedure instead of purely sequential procedure for the sake of simplicity and economy.

Notice that the result  $nY_n \rightarrow \frac{a^2\theta^2}{k}$  a.s. obtained by Jokiel-Rokita (2011) and  $\bar{S}_n \rightarrow \frac{k}{2\theta}$  a.s. assured by the strong law of large numbers. Now we describe the two-stage procedure. The procedure, by means of the definition of  $N_c$ , takes an initial sample of size  $n_0 = n_0(c) = [\delta c^{-\gamma}] + 1$  for some  $\delta > 0$  and for some  $0 < \gamma < \frac{1}{2}$ , a second sample to bring the sample size to

$$T_c = \max \left\{ n_0, \left\lceil \frac{|a|\sqrt{k}}{2\sqrt{c}\bar{S}_{n_0}} \right\rceil + 1 \right\}.$$

Then  $\theta$  can be estimated by  $\hat{\theta}_{T_c} = k/(2(\bar{S}_{T_c} + b_{T_c}))$ , where  $\bar{S}_{T_c} = \frac{1}{T_c} \sum_{i=1}^{T_c} s(X_i)$  and  $b_{T_c} = \frac{b}{T_c}$  with a fixed constant  $b > 0$ . Here the proposed two-stage procedure  $(T_c, \hat{\theta}_{T_c})$  is robust in the sense that it does not depend on the parameters of the gamma prior.

Let the posterior risk of the estimator  $\hat{\theta}_n$  be

$$\begin{aligned} Y_n^* &= E(L(\theta, \hat{\theta}_n) | \mathcal{F}_n) \\ &= \left( \frac{\lambda_n}{a + \lambda_n} \right)^{\alpha_n} e^{ak/(2(\bar{S}_n + b_n))} - \frac{ak}{2(\bar{S}_n + b_n)} + \frac{a\alpha_n}{\lambda_n} - 1, \end{aligned}$$

where  $b_n = \frac{b}{n}$  for all  $n \geq 1$ . Then the performance of the two-stage procedure  $(T_c, \hat{\theta}_{T_c})$  will be measured by its Bayes risk

$$R(T_c, \hat{\theta}_{T_c}) = E(L(\theta, \hat{\theta}_{T_c}) + cT_c) = E(Y_{T_c}^* + cT_c).$$

The family of the two-stage stopping times  $\{T_c; c > 0\}$  and the two-stage procedure  $(T_c, \hat{\theta}_{T_c})$  are APO and AO for a large class of gamma prior distributions in the following Theorem 2.1 and Theorem 2.2, respectively. The proofs for the two main theorems will be given in Section 3.

**Theorem 2.1.** (i)  $\{T_c; c > 0\}$  is APO with respect to  $\{Y_n + cn; n \geq 1\}$  and  $\{Y_n^* + cn; n \geq 1\}$ .  
(ii)  $\frac{Y_{T_c}^* + cT_c}{Y_{N_c} + cN_c} \rightarrow 1$  a.s. as  $c \rightarrow 0$ .

**Theorem 2.2.** If either  $-\lambda < a < 0$  or  $0 < a < b, \alpha \geq 1, \lambda > a$ , then the Bayes risk of the two-stage procedure  $(T_c, \hat{\theta}_{T_c})$  is

$$\begin{aligned} R(T_c, \hat{\theta}_{T_c}) &= \inf_T E\{L(\theta, \tilde{\theta}_T) + cT\} + o(\sqrt{c}) \\ &= \frac{2|a|\alpha}{\sqrt{k}\lambda} \sqrt{c} + o(\sqrt{c}) \quad \text{as } c \rightarrow 0, \end{aligned}$$

where the infimum extends over all  $\mathcal{F}_n$ -stopping times  $T$ .

### 3. Proof

In order to prove Theorem 2.1 and Theorem 2.2, we will develop some auxiliary results, whereas the proofs of the lemmas will be omitted in here.

**Lemma 3.1.** We have  $\sqrt{c}T_c \rightarrow \frac{|a|\theta}{\sqrt{k}}$  a.s. as  $c \rightarrow 0$ .

**Lemma 3.2.** We have  $nY_n^* \rightarrow \frac{a^2\theta^2}{k}$  a.s. as  $n \rightarrow \infty$ .

**Lemma 3.3.** For any given  $p > 1$ , there exists an integrable random variable that dominates  $(\sqrt{c}T_c)^p$  for all sufficiently small  $c$ .

**Lemma 3.4.** For any given  $p > 1$ , there exists an integrable random variable that dominates  $(\theta\bar{S}_{T_c})^{-p}$  for all sufficiently small  $c$ .

**Lemma 3.5.** For any  $p > 1$ ,  $\left\{ \left( \frac{\sqrt{c}T_c}{\theta} \right)^{-p}; c > 0 \right\}$  is uniformly integrable.

**Lemma 3.6.** If  $p > 0$  and either the case  $a < 0$  and  $\lambda + ap > 0$  or the other case  $a > 0, \alpha \geq 1$  and  $\lambda > ap \cdot \max\{1, \frac{\lambda}{b}\}$ , then  $\{e^{p\eta_{T_c}}; c > 0\}$  is uniformly integrable, where  $\eta_{T_c}$  is between 0 and  $a(k/(2(\bar{S}_{T_c} + b_{T_c})) - \theta)$ .

#### Proof of Theorem 2.1.

It follows from the definition of the APO rule  $N_c$  and the result  $nY_n \rightarrow \frac{a^2\theta^2}{k}$  a.s. in Joki-Rokita (2011) that  $cN_c^2 \rightarrow \frac{a^2\theta^2}{k}$  a.s. Hence, by Lemma 3.1, we have  $\frac{T_c}{N_c} \rightarrow 1$  a.s. Then, by the Remark of Theorem 2.1 in Bickel and Yahav (1967), we obtain  $\{T_c; c > 0\}$  is APO with respect to  $\{Y_n + cn; n \geq 1\}$ .



Using the fact that  $Y_n^* \geq Y_n$  a.s., we obtain the following inequalities

$$1 \leq \frac{Y_{T_c}^* + cT_c}{\inf_n(Y_n^* + cn)} \leq \frac{Y_{N_c} + cN_c}{\inf_n(Y_n + cn)} \cdot \frac{Y_{T_c}^* + cT_c}{Y_{N_c} + cN_c}.$$

Hence, by Lemmas 3.1 and 3.2, we have

$$\frac{Y_{T_c}^* + cT_c}{Y_{N_c} + cN_c} = \frac{T_c Y_{T_c}^* + cT_c^2}{N_c Y_{N_c} + cN_c^2} \cdot \frac{N_c}{T_c} \rightarrow 1 \text{ a.s.}$$

Then, by the results of the APO rule  $N_c$ ,

$$\frac{Y_{T_c}^* + cT_c}{\inf_n(Y_n^* + cn)} \rightarrow 1 \text{ a.s.,}$$

that is,  $\{T_c; c > 0\}$  is APO with respect to  $\{Y_n^* + cn; n \geq 1\}$ . The part (i) thus follows, and the proof of the part (ii) is also complete.  $\square$

### Proof of Theorem 2.2.

It follows from Lemmas 3.1 and 3.3 that

$$\begin{aligned} E(cT_c) &= \sqrt{c} \frac{|a|}{\sqrt{k}} E\theta + o(\sqrt{c}) \\ &= \frac{|a|}{\sqrt{k}} \frac{\alpha}{\lambda} \sqrt{c} + o(\sqrt{c}). \end{aligned}$$

Using Taylor's theorem, we obtain

$$\begin{aligned} \frac{1}{\sqrt{c}} L(\theta, \hat{\theta}_{T_c}) &= \frac{1}{\sqrt{c}} e^{\eta_{T_c}} \frac{a^2}{2} \left( \frac{k}{2(\bar{S}_{T_c} + b_{T_c})} - \theta \right)^2 \\ &= e^{\eta_{T_c}} \frac{a^2 k}{4\sqrt{c} T_c (\bar{S}_{T_c} + b_{T_c})^2} \left( \frac{\sqrt{T_c} (\bar{S}_{T_c} + b_{T_c} - \frac{k}{2\theta})}{\sqrt{\frac{k}{2\theta^2}}} \right)^2, \end{aligned}$$

where  $\eta_{T_c}$  is between 0 and  $a(k/(2(\bar{S}_{T_c} + b_{T_c})) - \theta)$ . It follows from the fact  $\eta_{T_c} \rightarrow 0$  a.s., Anscombe's theorem and Slutsky's theorem that

$$\frac{1}{\sqrt{c}} L(\theta, \hat{\theta}_{T_c}) \xrightarrow{D} \frac{|a|}{\sqrt{k}} G,$$

where  $G$  is defined by  $G(y) = EF_{\chi_1^2}(\frac{y}{\theta})$  for all  $y \in R$ , and  $F_{\chi_1^2}$  denotes the chi-squared distribution function with one degree of freedom.

On the other hand, we can rewrite

$$\begin{aligned} \frac{1}{\sqrt{c}}L(\theta, \hat{\theta}_{T_c}) &= e^{\eta_{T_c}} \frac{a^2\theta^2}{2\sqrt{c}(\bar{S}_{T_c} + b_{T_c})^2} \left( \bar{S}_{T_c} + b_{T_c} - \frac{k}{2\theta} \right)^2 \\ &\leq O(1)e^{\eta_{T_c}} \left( \frac{1}{\theta\bar{S}_{T_c}} \right)^2 \left( \frac{\theta}{\sqrt{cT_c}} \right)^2 \left\{ \sqrt{c}\theta^2 + \left( c^{\frac{1}{4}} \sum_{i=1}^{T_c} \frac{s(X_i) - \frac{k}{2\theta}}{\sqrt{\frac{k}{2\theta^2}}} \right)^2 \right\}. \end{aligned}$$

It follows from Lemma 2.3 of Hwang (1999) and Lemma 3.3 that for all sufficiently small  $c$ ,  $\left( c^{\frac{1}{4}} \sum_{i=1}^{T_c} \frac{s(X_i) - \frac{k}{2\theta}}{\sqrt{\frac{k}{2\theta^2}}} \right)^p$  is uniformly integrable for any given  $p \geq 2$ . Together with Lemmas 3.4, 3.5 and 3.6, we obtain for all sufficiently small  $c$ ,  $\frac{1}{\sqrt{c}}L(\theta, \hat{\theta}_{T_c})$  is uniformly integrable.

The conditions are needed here. Hence we have

$$\begin{aligned} EL(\theta, \hat{\theta}_{T_c}) &= \sqrt{c} \frac{|a|}{\sqrt{k}} E\theta + o(\sqrt{c}) \\ &= \frac{|a|}{\sqrt{k}} \frac{\alpha}{\lambda} \sqrt{c} + o(\sqrt{c}). \end{aligned}$$

The proof is thus complete.  $\square$

## Acknowledgements

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# 國科會補助計畫衍生研發成果推廣資料表

日期:2012/10/05

國科會補助計畫	計畫名稱: 二階段抽樣法在非對稱損失函數下貝氏序列估計之研究
	計畫主持人: 黃連成
	計畫編號: 100-2118-M-029-001- 學門領域: 統計推論
無研發成果推廣資料	

100 年度專題研究計畫研究成果彙整表

計畫主持人：黃連成		計畫編號：100-2118-M-029-001-				計畫名稱：二階段抽樣法在非對稱損失函數下貝氏序列估計之研究	
成果項目		量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）	
		實際已達成數（被接受或已發表）	預期總達成數（含實際已達成數）	本計畫實際貢獻百分比			
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	1	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

<p>其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)</p>	<p>無</p>
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	成果項目	量化	名稱或內容性質簡述
科 教 處 計 畫 加 填 項 目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與(閱聽)人數	0	

# 國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

達成目標

未達成目標（請說明，以 100 字為限）

實驗失敗

因故實驗中斷

其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文： 已發表  未發表之文稿  撰寫中  無

專利： 已獲得  申請中  無

技轉： 已技轉  洽談中  無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）