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通貨膨脹與通貨膨脹不確定性的非線性關係：跨國實證研究

The Nonlinear Relationship between Inflation and Inflation Uncertainty: Evidence of East Asian Countries

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Abstract

We revisit the relationship between inflation and inflation uncertainty by a nonlinear flexible regression model of four economies in East Asia, that is, Taiwan, Hong Kong, Singapore, and South Korea. Two hypothesis will be examined. One hypothesis is proposed by Friedman (1977). He argued that increased inflation could raise inflation uncertainty. The other hypothesis is provided by Cukierman and Meltzer (1986), they argued that high level of inflation uncertainty will cause higher rate of inflation. We find overwhelming statistical evidences in favor of Friedman's hypothesis except for Hong Kong. The nonlinearity displays an U shape pattern, implying that high rate of inflation or deflation will cause high inflation uncertainty. On the other hand, Cukierman-Meltzer's hypothesis is also evidenced for all four economies. Hong Kong, Singapore, and South Korea have a positive relation in favor of Cukierman-Meltzer's hypothesis, while Taiwan has an inverted-U shape.

Key Words: Inflation, Inflation Uncertainty, Nonlinear, Flexible Regression Model

JEL Classification: C22, E31

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Chapter 1

Introduction

Inflation is always an important issue in economics. Many economists try to explore what inflation matters. A well-known Philips curve explains the negative relationship between inflation and economic output.

Okun (1971) was the first to argue that inflation is positively associated with inflation uncertainty. He found that countries experience higher inflation rate will have larger standard deviation of inflation. Friedman (1977) also argued that, in his Nobel address, higher rate of inflation invokes higher inflation uncertainty. High inflation uncertainty will reduce economic efficiency via the distortion of price signal, such distortions may exert negative impacts on the efficiency of resource allocation and the level of real economic activity. Ball (1992) formalized Friedman's hypothesis and provided a theoretical foundation for the positive relationship between inflation and inflation uncertainty. In his model, there are two type of policymakers who stochastically alternate in power, and the public knows that only one type is willing to bear the economic costs of disinflation. During periods of low inflation, the monetary authorities are willing to keep it low to lower inflation uncertainty. On the contrary, during periods of high inflation, the public does not know for how long it will last before an anti-inflation policy makers come in power.

On the other hand, Cukierman and Meltzer (1986) claimed a reverse direction which is contrast to Friedman's hypothesis that higher inflation uncertainty will raise the rate of inflation. In their model, in the absence of a commitment mechanism, the monetary authorities may engage in discretionary policy. Therefore, the public becomes uncertain about the monetary policy, there is an incentive for the central bankers to act *opportunistically* in terms of seeking to attain higher short-term economic growth. Therefore higher inflation uncertainty will raise average inflation rate. On the contrary, Holland (1995) argued that, under assumptions of Friedman's hypothesis hold and negative effect of inflation uncertainty on growth, the monetary authorities have a motive to *stabilize* inflation when uncertainty rises.

Many empirical studies provide mixing conclusions about this issue. For example, Grier and Perry (1998) investigated the linkage between inflation and inflation uncertainty in the G7 countries. They provided evidences in favor of Friedman's hypothesis for all G7 countries. On the other hand, Japan and France are in favor of Cukierman-Meltzer's hypothesis, while for the U.S., UK and Germany, Holland's hypothesis is accepted. Tevfik and Perry (2000) also found strong evidences in favor of Friedman's hypothesis for Turkey. But the evidence of Cukierman and Meltzer's hypothesis is mixed. Fountas (2001) used a long series of UK inflation data and provided strong evidence in favor of the hypothesis that inflationary periods are associated with high inflation uncertainty, and also indicated that more inflation uncertainty leads to lower output.

Engle (1982) and Bollerslev (1986) estimated ARCH and GARCH model for the U.S. inflation and found that higher rate of inflation does not essentially imply higher inflation uncertainty. Ball and Cecchetti (1990) investigated the relationship between inflation and inflation uncertainty in US at short and long horizon. They decomposed the movements in inflation into shifts in trend inflation and temporary deviations from trend, and employed the variance of deviation from the trend as a proxy of uncertainty measurement. Their empirical results found that a rise in the level of inflation has only little effect on the variance of deviations. They also found that inflation had much larger effects on uncertainty at long horizons.

Cosimano and Jansen (1988) also fitted the ARCH model for the U.S. inflation series and found that Friedman's hypothesis was rejected. Baillie et al. (1996) examined ten countries by ARFIMA-GARCH model where the fractionally integrated process allows to provide more insight for macroeconomists on the persistence of shocks. They found only three high-inflation countries (Argentina, Brazil, and Israel) are in favor of Cukierman-Meltzer's hypothesis. They also provided the clear evidences for the G7 countries that although inflation shocks have long memory they are nevertheless mean reverting.

Hua (2000) examined the linkage between inflation and inflation uncertainty in six countries in Asia. All six countries are intended to accept Friedman's hypothesis. She also found that inflation uncertainty causes inflation in other five countries (Philippines, South Korea, Thailand, Malaysia, and Singapore) except for Taiwan. Chiu (2003) investigated the interrelationships between inflation, inflation uncertainty, and output growth in Taiwan using bivariate-GARCH model. She provided the evidence that current and lags of inflation all have positive influence on inflation uncertainty. An increase in real output growth uncertainty is evident to lead to an increase in both output growth and inflation.

In spite of aboundant literature on the inflation and inflation uncertainty, most of them are based on the GARCH-type model and a shortcoming of this model is that it extracts only *linear* relationship between inflation and inflation uncertainty. It overlooks the *nonlinear* relationship if it really exit in the data. We have no reasons to exclude, whatsoever, other possible functional forms for describing such a relation. In this study, we revisit Friedman's and Cukierman and Meltzer's hypotheses in terms of Hamilton (2001) flexible regression model. The merit of this approach is that we can simultaneously detect linear and nonlinear relationships of the data.

Within the flexible nonlinear inference, the nonlinear tests are based on the Largrange-multiplier test. The null hypothesis is absence of nonlinearity, while the alternative hypothesis allows for a broad class of deterministic nonlinear function. Following Hamilton (2001), Dahl and Gonzále-Rivera (2003) also developed various nonlinear test statistic.

We use monthly data of the consumer price index (CPI) of four East Asian economies, that is, Taiwan, Hong Kong, Singapore, and South Korea. In our empirical results, we find that Taiwan, Singapore, and South Korea are in favor of Friedman's hypothesis while Hong Kong fails to support it. In details, the patterns of the effect of inflation on inflation uncertainty show the U shape in these three economies. The nonlinear patterns suggest that inflation uncertainty appear to increase in inflationary and deflationary period.

Moreover, these four economies all accept Cukierman-Meltzer's hypothesis, implying that high level of inflation uncertainty will raise higher rate of inflation. Hong Kong, Singapore, and South Korea show a absolutely positive and nonlinear relation in favor of Cukierman-Meltzer's hypothesis. The results imply that the central banker of these economies are intended to behave opportunistic policies to create inflation surprises to rise economic output. Interestingly, Taiwan has inverted-U relation of the effect of inflation uncertainty on inflation. It implies that the monetary authorities only act opportunistic policy under a specific level of inflation uncertainty.

The outline of the paper is as follows: In Chapter 2, we will review the flexible regression model and nonlinear test of the model. Chapter 3 presents our empirical results of four Asian economies. Conclusion are offered in Chapter 4.

Chapter 2

Flexible Regression Model

Hamilton (2001) proposed a new approach, flexible regression model, to detect the nonlinearity of the data. He employed the concept of random field to describe the nonlinear component of the model. Consider the following econometric model

$$y_t = \mu(\boldsymbol{x}_t) + \varepsilon_t, \tag{2.1}$$

where

$$\mu(\boldsymbol{x}_t) = \alpha_0 + \boldsymbol{\alpha}'\boldsymbol{x}_t + \lambda m(\boldsymbol{g} \odot \boldsymbol{x}_t). \tag{2.2}$$

for y_t and $\boldsymbol{x}_t$ are stationary and ergodic process. In this model, the symbol $\odot$ denotes the element-by-element multiplication, and $m(\cdot)$ is outcome of the random field. In equation (2.1), it contains the linear component $\alpha_0 + \boldsymbol{\alpha}'\boldsymbol{x}_t$ and the nonlinear component $\lambda m(\boldsymbol{g} \odot \boldsymbol{x}_t)$, where $m(\cdot)$ is latent and unseen. Term λ makes a contribution to the nonlinearity and $\boldsymbol{g}$ controls the curvature.

In section 2.1, we introduce the concept of random field. Section 2.2 describes how to infer the conditional mean $\mathrm{E}(y_t \mid \boldsymbol{x}_t) = \mu(\boldsymbol{x}_t)$. Section 2.3 discusses the associated nonlinear tests.

2.1 Random Field

2.1.1 Euclidean Distance

We divide this section into two part. The former is on single explanatory variable case, and this will give us clear concept on the random field. The latter, we extend the model into k explanatory variables, this could generalize mathematical expressions.

Single Explanatory Variable

We first describe a latent stochastic process $m(x)$ which will be used to characterize the conditional expectation function $\mu(x)$. Consider an interval $[a, b]$ in $\Re^1$. ω is a parameter described shortly, and partition the interval $[a-\omega, b+\omega]$ as $\{x_1, x_2, \ldots, x_N\}$, where $x_1 = a - \omega$, $x_N = b + \omega$, and $x_i = x_{i-1} + \Delta_N$ for $i = 2, \ldots, N$. Each node x_i on the interval generates a Standard Normal variable $e(x_i)$ with mutual independence. Furthermore, for each node x_i such that $a \leqslant x_i \leqslant b$,[1] we construct a random variable $m_N(x_i)$, called *random field*, which is the proportionality of the value of $e(x_i)$ summed for all x_j whose distance from x_i is less than or equal to ω to the square root of the number of $e(x_i)$. That is

$$m_N(x_i) = (1 + 2\omega/\Delta_N)^{-1/2} \sum_{j=-\omega/\Delta_N}^{\omega/\Delta_N} e(x_{i+j}), \tag{2.3}$$

where $(1 + 2\omega/\Delta_N)$ could be the number of $e(x_i)$, and $m_N(x_i) \sim \mathcal{N}(0, 1)$ with moving average representation. An example is given below in Example 1 and illustrates in Figure 2.1 and Figure 2.2.

Example 1. In the case of $\omega = 1$, $\Delta_N = 1$, and $N = 12$. Consider an interval $[a, b]$ in $\Re^1$ and partition the interval as $\{x_1, x_2, \ldots, x_{12}\}$ where $x_i = x_{i-1}+1$ for $i = 2, \ldots, 12$. Equation (2.3) can be written as

$$m(x_i) = \frac{1}{\sqrt{3}} \sum_{j=-1}^{1} e(x_{i-j})$$

for $a \leq x_i \leq b$. For instance,

$$m(x_2) = \frac{1}{\sqrt{3}} [e(x_1) + e(x_2) + e(x_3)],$$

$$m(x_3) = \frac{1}{\sqrt{3}} [e(x_2) + e(x_3) + e(x_4)].$$

[1] For $x_i < a$ and $x_i > b$, $m(x_i)$ cannot be obtained.

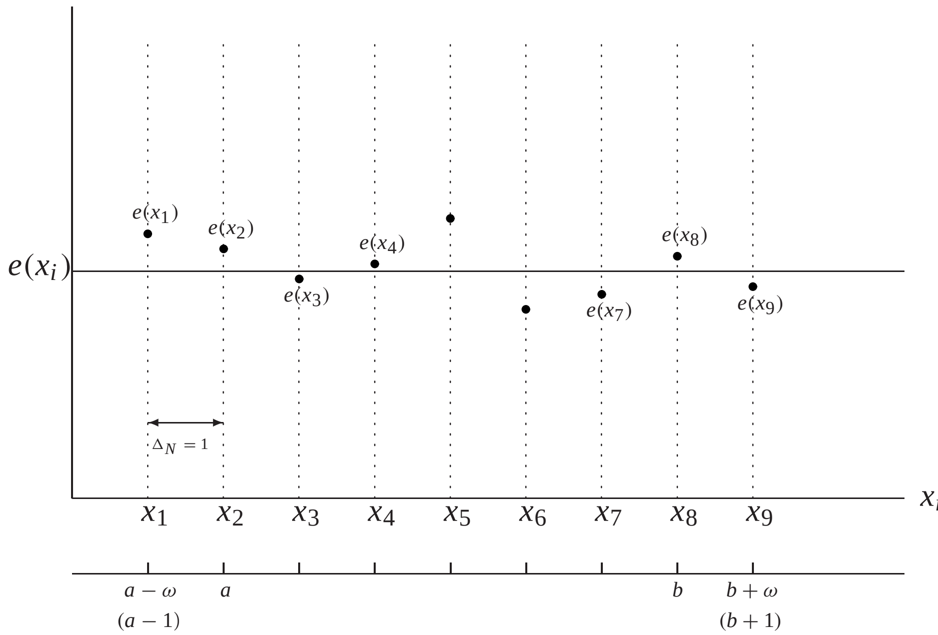


Figure 2.1: Illustration of interest $\{x_1, x_2, \ldots, x_9\}$ on interval $[a-\omega, b+\omega]$ in which nodes generates random variable $e(x_i)$ for $i = 1, \ldots, 9$ in case of $\omega = 1$ and $\Delta_N = 1$.

As for the covariance (correlation) between $m(x_i)$ and $m(x_j)$, it is

$$\mathrm{E}[m(x_i)m(x_j)] = \begin{cases} 1 - |x_i - x_j|/2\omega & \text{if } |x_i - x_j| \leqslant 2\omega, \\ 0 & \text{otherwise.} \end{cases} \tag{2.4}$$

In the case of example 1, the correlation $\mathrm{E}[m(x_i)m(x_j)]$ is $1/2$. In detail, it is the ratio of the volume of the overlap of the unit-distance lines to the unit-distance line. As shown in Figure 2.2, the correlation between any pair of nodes is, for example, as follows

$$\mathrm{E}[m(x_2)m(x_3)] = \overline{BC}/\overline{AB} = 1/2,$$

or

$$\mathrm{E}[m(x_7)m(x_8)] = \overline{EF}/\overline{DF} = 1/2.$$

In continuous condition, $N \to 0$ and $\Delta_N \to 0$, a single realization of this process associates each $x \in [a, b]$ with a value $m(x) \in \Re^1$. We can characterize the function as

$$m(x) = (2\omega)^{-1/2}[W(x+\omega) - W(x-\omega)], \tag{2.5}$$

where $W(\cdot)$ is a standard Wiener process. Note that any given realization of $m(\cdot)$ is continuous in x but not differentiable.

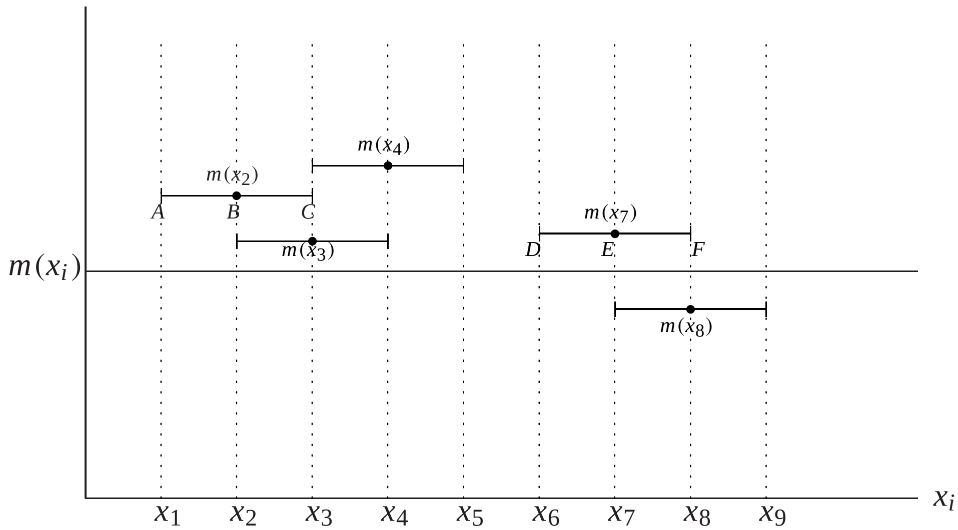


Figure 2.2: Illustration of the correlation between random fields. It is the ratio of the volume of the overlap of unit-distance lines to the unit-distance line. The case of $\omega = 1$ and $\Delta_N = 1$.

K Explanatory Variables

We here extend single explanatory variable to K explanatory variables. Analogous to the previous section, define a grid in $\Re^k$ by the nodes $\{\boldsymbol{x}(i_1, i_2, \ldots, i_k)\}$ where the index $i_j \in \{1, \ldots, N\}$ for $j = 1, \ldots, k$. Let A_N be the set consisting of the N^k distinct points in $\Re^k$ covered by this grid. For each $\boldsymbol{x} \in A_N$ generates random variable $e(\boldsymbol{x}) \sim \mathcal{N}(0, 1)$ which is independent of $e(\boldsymbol{z})$ for $\boldsymbol{x} \neq \boldsymbol{z}$. In similar fashion, define $B_N \subset A_N$ which the Euclidean distance from $\boldsymbol{x}$ is less or equal to one [2]

$$B_N(\boldsymbol{x}) = \{\boldsymbol{z} \in A_N : (\boldsymbol{x} - \boldsymbol{z})'(\boldsymbol{x} - \boldsymbol{z}) \leqslant 1\}. \tag{2.6}$$

To reiterate (2.3) in the same way, we can obtain

$$m_N(\boldsymbol{x}) = [n_N(\boldsymbol{x})]^{-1/2} \sum_{\boldsymbol{z} \in B_N(\boldsymbol{x})} e(\boldsymbol{z})$$

where $n_N(\boldsymbol{x})$ indicates the number of points in $B_N(\boldsymbol{x})$. Undoubtedly, $m(\boldsymbol{x})$, continuous form, is also distributed $\mathcal{N}(0, 1)$ with moving average representation.

As for the correlation $\mathrm{E}[m(\boldsymbol{x})m(\boldsymbol{z})] = H_k(h)$, it exhibits as

$$H_k(h) = \begin{cases} G_{k-1}(h, 1)/G_{k-1}(0, 1) & \text{if } h \leqslant 1, \\ 0 & \text{if } h > 1, \end{cases} \tag{2.7}$$

[2] We normalize the distance parameter $\omega = 1$ for convenience.

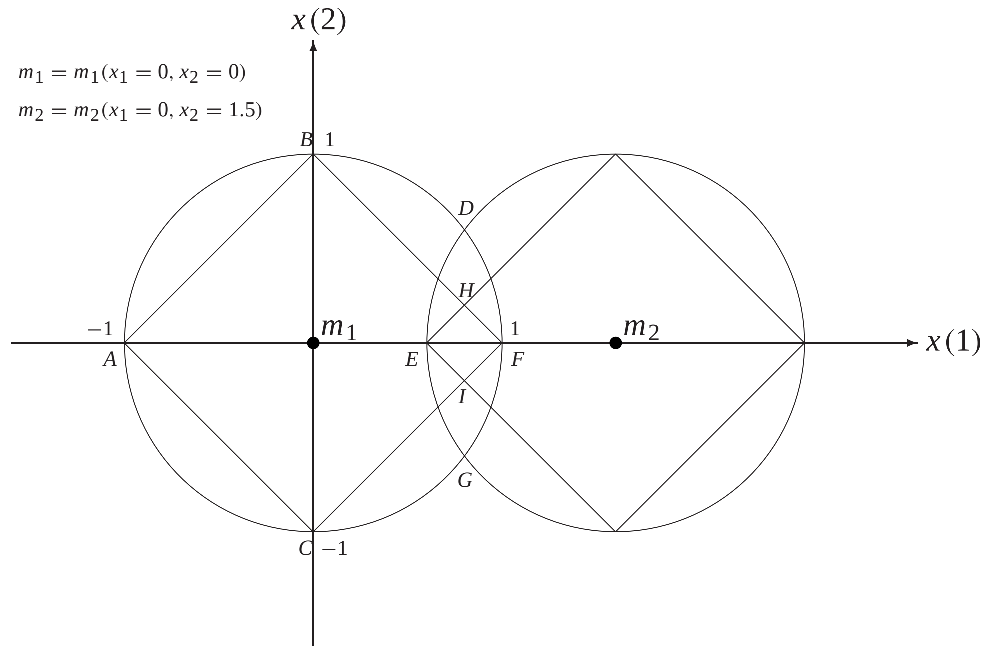


Figure 2.3: Illustration of the covariances based L_1 norm and L_2 norm respectively in the case of $k = 2$.

where

$$G_k(h, r) = \int_h^r (r^2 - z^2)^{k/2}\, dz$$

for $r = 1, h \equiv (1/2)[(\boldsymbol{x} - \boldsymbol{z})'(\boldsymbol{x} - \boldsymbol{z})]^{1/2}$ based on Euclidean distance, and dimensionality k. Precise to say, it is the ratio of the volume of the overlap of k-dimensional unit spheroids centered at $\boldsymbol{x}$ and $\boldsymbol{z}$ to the volume of a single k-dimensional unit spheroid. Figure 2.3 demonstrates that $k = 2$, and the correlation between m_1 and m_2 can denote as

$$\mathrm{E}[m_1 m_2] = \frac{\text{Superficial area of } DEFG}{\text{Superficial area of unit shperiods}}.$$

2.1.2 Minkowski Distance

In Appendix A, we clarify the difference between Euclidean and Minkowski distance. Here we discuss the random field under Manhattan distance (or L_1 norm). Analogous to the equation (2.6), under the L_1 norm we redefine the set B_N^* denoted as

$$B_N^*(\boldsymbol{x}) = \{\boldsymbol{z} \in A_N : |\boldsymbol{x} - \boldsymbol{z}|'\mathbf{1} \leqslant 1\},$$

where $\mathbf{1}$ is a k-dimensional weighting coefficient vector consisting of ones in each entry and let $m_N^*(\boldsymbol{x})$ be the moving average representation

$$m_N^*(\boldsymbol{x}) = n_N^*(\boldsymbol{x})^{1/2} \sum_{\boldsymbol{z} \in B_N^*(\boldsymbol{x})} e(\boldsymbol{z}), \tag{2.8}$$

where $n_N^*(\boldsymbol{x})$ denotes the number of points in $B_N^*(\boldsymbol{x})$. Also $m^*(\boldsymbol{x})$ is the continuous form of the random field under L_1 norm. In the same way, the correlation H_k^* between $m^*(\boldsymbol{x})$ and $m^*(\boldsymbol{z})$ is given by the ratio of the volume of a k-dimensional unit *orthogons* centered at $\boldsymbol{x}$ and $\boldsymbol{z}$ to the volume of a k-dimensional unit orthogon. That is

$$H_k^*(h^*) = \begin{cases} G_{k-1}^*(h^*, 1)/G_{k-1}^*(0, 1) & \text{if } h^* \leqslant 1, \\ 0 & \text{if } h^* > 1, \end{cases} \tag{2.9}$$

where

$$G_k^*(h^*, r) = \int_{h^*}^{r} (r - |z|)^k \, dz$$

for $r = 1$ and $h^* = (1/2)|\boldsymbol{x} - \boldsymbol{z}|'\mathbf{1}$. Figure 2.3 shows the correlation of the random filed under L_1 norm and L_2 norm, respectively. Notice that in the case of $k = 1$ the L_2 norm and L_1 norm will provide identical covariance functions $\overline{EF}/\overline{AF}$, and in the case of $k = 2$ based on L_1 norm is

$$\mathrm{E}[m_1^* m_2^*] = \frac{\text{Orthogon area of } EFHI}{\text{Orthogon area of } ABCF}.$$

Differ from the L_2 norm, the disadvantage of the random field with a moving average representation in L_1 norm is that the field is not isotropic.[3] With that, Dahl and Gonzále-Rivera (2003) propose a permissible covariance function with isotropy in the basis of L_1 norm, that is

$$C_k^*(h^*) = \begin{cases} (1 - h^*)^{2k} & \text{if } h^* \leqslant 1, \\ 0 & \text{if } h^* > 1, \end{cases} \tag{2.10}$$

where C_k^* satisfies the positive semidefiniteness condition, that is $\boldsymbol{q}'C^*\boldsymbol{q}$, for all $\boldsymbol{q} \neq \mathbf{0}_T$, which ensures that the random field is homogenous. Another advantage of permissible

[3] A scalar random field is said to be Gaussian and it is completely determined by its mean function $\mu(\boldsymbol{x}) = \mathrm{E}[m(\boldsymbol{x})]$ and its covariance function with typical element $\boldsymbol{C}(\boldsymbol{x}, \boldsymbol{z}) = \mathrm{E}[(m(\boldsymbol{x}) - \mu(\boldsymbol{x}))(m(\boldsymbol{z}) - \mu(\boldsymbol{z}))]$. The random field is said to be homogenous or stationary if $\mu(\boldsymbol{x}) = \mu$ and the covariance function depends only on the difference vector $\boldsymbol{x} - \boldsymbol{z}$ and we write $\boldsymbol{C}(\boldsymbol{x}, \boldsymbol{z}) = \boldsymbol{C}(\boldsymbol{x} - \boldsymbol{z})$. Furthermore, the random field is said to be isotropic if the covariance function depends on $d(\boldsymbol{x}, \boldsymbol{z})$, where $d(\cdot)$ is a scalar measure of distance.

covariance function is that we can firstly derive covariance function and then derive its corresponding random field due to the Khinchin (1934) theorem and Bochner (1959) theorem. On the contrary, Hamilton firstly proposes a moving average representation of the random field and derives its covariance function.

2.2 Inference of Conditional Mean

In equation (2.1), Hamilton uses a moving average representation random field to describe the nonlinearity of the data, where the random field is a Gaussian, homogenous, and isotropic. The goal, in this section,is to infer the condition mean $\mathrm{E}(y_t \mid \boldsymbol{x}_t) = \mu(\boldsymbol{x}_t)$ and describes the data's characteristics.

2.2.1 Maximum Likelihood Estimation

Let us imagine how we could form an 'optimal' inference about the unseen conditional expectation function $\mu(\boldsymbol{x}_t)$. Hamilton used a recursive formulation, like the Kalman filter, to accomplish the estimation work.

Consider the evaluating function $\mu(\boldsymbol{x}_t)$ with observed $\boldsymbol{x}_t = \boldsymbol{\tau}_1, \boldsymbol{\tau}_2, \ldots, \boldsymbol{\tau}_N$, such that

$$\boldsymbol{\mu} = \begin{bmatrix} \mu(\boldsymbol{\tau}_1) \\ \mu(\boldsymbol{\tau}_2) \\ \vdots \\ \mu(\boldsymbol{\tau}_N) \end{bmatrix}.$$

From equations (2.2) and (2.7), it follows that

$$\boldsymbol{\mu} \sim \mathcal{N}(\boldsymbol{\xi}_0, \boldsymbol{P}_0) \tag{2.11}$$

where the ith element of $\boldsymbol{\xi}_0$ is given by $\alpha_0 + \boldsymbol{\alpha}'\boldsymbol{\tau}_i$, and the row i, column j element of $\boldsymbol{P}_0$ is given by

$$p_{ij}^{(0)} = \begin{cases} \lambda^2 H_k(h_{ij}) & \text{if } h_{ij} < 1, \\ 0 & \text{if } h_{ij} \geq 1, \end{cases}$$

for

$$h_{ij} = \frac{1}{2}\{[(\boldsymbol{g} \odot \boldsymbol{\tau}_i) - (\boldsymbol{g} \odot \boldsymbol{\tau}_j)]'[(\boldsymbol{g} \odot \boldsymbol{\tau}_i) - (\boldsymbol{g} \odot \boldsymbol{\tau}_j)]\}^{1/2}.$$

We further consider a model

$$y_t = \boldsymbol{i}_t' \boldsymbol{\mu} + \varepsilon_t, \tag{2.12}$$

where $\boldsymbol{x}_t = \boldsymbol{\tau}_{j_t}$, $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ and $\boldsymbol{i}_t$ denotes column j_t of the $(N \times N)$ identity matrix. Given the earlier result (2.11), and then

$$y_1 \mid \boldsymbol{x}_1 \sim \mathcal{N}(\boldsymbol{i}_1' \boldsymbol{\xi}_0, \boldsymbol{i}_1' \boldsymbol{P}_0 \boldsymbol{i}_1 + \sigma^2),$$

so that[4]

$$\boldsymbol{\mu} \mid y_1, \boldsymbol{x}_1 \sim \mathcal{N}(\boldsymbol{\xi}_1, \boldsymbol{P}_1) \tag{2.13}$$

where

$$\begin{aligned} \boldsymbol{\xi}_1 &= \boldsymbol{\xi}_0 + \frac{\boldsymbol{P}_0 \boldsymbol{i}_1 (y_1 - \boldsymbol{i}_1' \boldsymbol{\xi}_0)}{\boldsymbol{i}_1' \boldsymbol{P}_0 \boldsymbol{i}_1 + \sigma^2}, \\ \boldsymbol{P}_1 &= \boldsymbol{P}_0 - \frac{\boldsymbol{P}_0 \boldsymbol{i}_1 \boldsymbol{i}_1' \boldsymbol{P}_0}{\boldsymbol{i}_1' \boldsymbol{P}_0 \boldsymbol{i}_1 + \sigma^2}. \end{aligned}$$

By the same token, we can infer $\boldsymbol{\mu}$ conditional on $\boldsymbol{Y}_{t-1} = (y_1, \boldsymbol{x}_1', y_2, \boldsymbol{x}_2', \ldots, y_{t-1}, \boldsymbol{x}_{t-1}')'$, that is

$$\boldsymbol{\mu} \mid \boldsymbol{Y}_{t-1} \sim \mathcal{N}(\boldsymbol{\xi}_{t-1}, \boldsymbol{P}_{t-1}). \tag{2.14}$$

We furthermore assume that $\boldsymbol{x}_t$ is exogenous to the $\mu(\cdot)$, so that

$$\boldsymbol{\mu} \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1} \sim \mathcal{N}(\boldsymbol{\xi}_{t-1}, \boldsymbol{P}_{t-1}). \tag{2.15}$$

By the principle of *iteration*, like (2.13) to (2.14) , we can get the result

$$\boldsymbol{\mu} \mid \boldsymbol{Y}_t \sim \mathcal{N}(\boldsymbol{\xi}_t, \boldsymbol{P}_t) \tag{2.16}$$

where

$$\begin{aligned} \boldsymbol{\xi}_t &= \boldsymbol{\xi}_{t-1} + \frac{\boldsymbol{P}_{t-1} \boldsymbol{i}_t (y_t - \boldsymbol{i}_t' \boldsymbol{\xi}_{t-1})}{\boldsymbol{i}_t' \boldsymbol{P}_{t-1} \boldsymbol{i}_t + \sigma^2}, \\ \boldsymbol{P}_t &= \boldsymbol{P}_{t-1} - \frac{\boldsymbol{P}_{t-1} \boldsymbol{i}_t \boldsymbol{i}_t' \boldsymbol{P}_t - 1}{\boldsymbol{i}_t' \boldsymbol{P}_{t-1} \boldsymbol{i}_t + \sigma^2}, \\ \boldsymbol{Y}_t &= (y_t, \boldsymbol{x}_t', y_{t-1}, \boldsymbol{x}_{t-1}', \ldots, y_1, \boldsymbol{x}_1')'. \end{aligned}$$

[4]For more detail, readers are referred to Hamilton (2001) Lemma 3.1 fro more details

From the above descriptions, Hamilton uses iteration method to derive the condition mean gradually. Finally, as show in (2.16), he uses all the data to derive the latent $\boldsymbol{\mu}$. But there still exists some unknown population parameters $(\alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda)'$ needed to be estimated. We can obtain them by MLE. It follows that from (2.12) and (2.15), and yields

$$y_t \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1} \sim \mathcal{N}(\boldsymbol{i}_t'\boldsymbol{\xi}_{t-1}, \boldsymbol{i}_t'\boldsymbol{P}_{t-1}\boldsymbol{i}_t + \sigma^2).$$

So, we can calculate the conditional log-likelihood of the tth observation from

$$\ln f(y_t \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1}; \alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\boldsymbol{i}_t'\boldsymbol{P}_{t-1}\boldsymbol{i}_t + \sigma^2) - \frac{1}{2}\frac{(y_t - \boldsymbol{i}_t'\boldsymbol{\xi}_{t-1})^2}{\boldsymbol{i}_t'\boldsymbol{P}_{t-1}\boldsymbol{i}_t + \sigma^2},$$

and estimate the unknown population parameters $(\alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda)'$ by maximizing

$$\sum_{t=1}^{T} \ln f(y_t \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1}; \alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda). \tag{2.17}$$

Thus, we can get $(\hat{\alpha}, \hat{\boldsymbol{\alpha}}', \hat{\sigma}, \hat{\boldsymbol{g}}', \hat{\lambda})'$ and then infer the estimated conditional mean $\hat{\boldsymbol{\mu}}$.

2.2.2 GLS Representation

We can not infer the conditional mean $\boldsymbol{\mu}$ and the unknown parameters $(\alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda)'$ since the random field $m(\cdot)$ is unseen and latent. So Hamilton represents the model as GLS form to circumvent this problem. In other word, we separate unknown part into residual. That is to say, we represent it as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{x}_1' \\ 1 & \boldsymbol{x}_2' \\ \vdots & \vdots \\ 1 & \boldsymbol{x}_T' \end{bmatrix} \begin{bmatrix} \alpha_0 & \boldsymbol{\alpha}' \end{bmatrix} + \begin{bmatrix} \lambda m(\boldsymbol{g} \odot \boldsymbol{x}_1) + \varepsilon_1 \\ \lambda m(\boldsymbol{g} \odot \boldsymbol{x}_2) + \varepsilon_2 \\ \vdots \\ \lambda m(\boldsymbol{g} \odot \boldsymbol{x}_T) + \varepsilon_T \end{bmatrix}.$$

To simplify it as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}, \tag{2.18}$$

where $\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{X}\boldsymbol{\beta}, \boldsymbol{P}_0 + \sigma^2\boldsymbol{I}_T)$, and $\boldsymbol{u}$ is the residual, the part of nonlinear and latent component, which helps us to avoid confirming the unknown stochastic process $m(\cdot)$.

For more details, when estimating $(\alpha, \boldsymbol{\alpha}', \sigma, \boldsymbol{g}', \lambda)'$, we first classify the model (2.1) into two component: linear (known) component $\alpha_0 + \boldsymbol{\alpha}_t'$, and nonlinear (unknown) component $\lambda m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t$. Second, we rewrite it as GLS form (2.18). Third, we calculate the log likelihood of $\boldsymbol{y}$:

$$\begin{aligned}\ln f(\boldsymbol{y}; \boldsymbol{\psi}, \boldsymbol{\theta}) = &-\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln\sigma^2 - \frac{1}{2}\ln|\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})| \\ &- \frac{1}{2\sigma^2}(\boldsymbol{y} - \boldsymbol{X\beta})'\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1}(\boldsymbol{y} - \boldsymbol{X\beta})\end{aligned}$$

where

$$\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta}) \equiv \zeta^2 \boldsymbol{H}(\boldsymbol{g}) + \boldsymbol{I}_T$$

for $\zeta \equiv \lambda/\sigma$, linear parameters $\boldsymbol{\psi} = (\alpha_0, \boldsymbol{\alpha}', \sigma^2)$, nonlinear parameters $\boldsymbol{\theta} = (\boldsymbol{g}', \zeta)'$, and $\boldsymbol{H}(\boldsymbol{g})$ denoted as $(T \times T)$ matrix of $H_k(\cdot)$ in (2.7). Given the nonlinear part $\boldsymbol{\theta}$, we can obtain the linear part $\boldsymbol{\psi}$ of the model consisting of $\boldsymbol{\beta}$ and σ^2 as

$$\tilde{\boldsymbol{\beta}}(\boldsymbol{\theta}) = [\boldsymbol{X}'\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1}\boldsymbol{X}]^{-1}[\boldsymbol{X}'\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1}\boldsymbol{y}], \tag{2.19}$$

$$\tilde{\sigma}^2(\boldsymbol{\theta}) = [\boldsymbol{y} - \boldsymbol{X}\tilde{\boldsymbol{\beta}}(\boldsymbol{\theta})]'\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1}[\boldsymbol{y} - \boldsymbol{X}\tilde{\boldsymbol{\beta}}(\boldsymbol{\theta})]/T. \tag{2.20}$$

Thus we can write the concentrated log likelihood function as

$$\eta(\boldsymbol{\theta}; \boldsymbol{y}, \boldsymbol{X}) = -\frac{T}{2}\ln(2\pi) - \frac{T}{2}\ln\tilde{\sigma}^2(\boldsymbol{\theta}) - \frac{1}{2}\ln|\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})| - \frac{T}{2} \tag{2.21}$$

Given the equation (2.21), we can estimate nonlinear component $\hat{\boldsymbol{\theta}}$ by maximizing it. Inserting $\hat{\boldsymbol{\theta}}$ into (2.19) and (2.20) to get $\hat{\boldsymbol{\psi}}$.

Hamilton estimates the unknown population parameters $\boldsymbol{\psi}$ and $\boldsymbol{\theta}$ in different way from subsection 2.2.1. Indeed, it circumvents the problem of unseen stochastic process $m(\cdot)$, and instead he uses GLS representation to separate it into the *residual* which helps us to simplify the complexity of calculations.

2.3 Nonlinearity Test

In the framework of (2.1) and (2.2), it is easy to observed that we can test the linearity either by λ or vector $\boldsymbol{g}$, which makes contribution to the nonlinearity and curvature, respectively. Dahl and Gonzále-Rivera (2003) denote them as λ-test and g-test. When proceeding the test, there exists a nuisance parameter problem, where a set of parameter are unidentified under the null hypothesis. For more details: (i) In the case of null hypothesis $H_0 : \lambda^2 = 0$, the parameter vector $\boldsymbol{g}$ will become unidentified under the null and the number of unidentified parameters increases with the dimensionality of the model. (ii) In the case of null hypothesis $H_0 : \boldsymbol{g} = \mathbf{0}_k$, only one λ will become unidentified whatever dimensionality of model increases, and the stochastic process becomes *nonergodic*, where the ergodicity is critical assumption for the *law of the large number* to hold. Nonergodicity imply that the nonlinearity test based on $\boldsymbol{g}$ may not have a well defined asymptotic distribution under the null. Hence Hamilton deals with the problem in the first case by fixing $\boldsymbol{g}$, and Dahl and Gonzále-Rivera (2003) redefine the random field $m(\boldsymbol{x})$ as Minikoski distance instead of Euclidean distance to circumvent the nuisance problem. We here rewrite a general function of (2.18), that is

$$\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{X\beta}, \lambda^2 \boldsymbol{C}_k + \sigma^2 \boldsymbol{I}_T), \tag{2.22}$$

where $\boldsymbol{C}_k$ is generic covariance function determines its corresponding random field uniquely. $\boldsymbol{C}_k$ can be Hamilton's in (2.7) based on L_2 norm, or be that of Dahl and González-Rivera's in (2.9) based on L_1 norm, or be permissible covariance in (2.10). Further we can calculate the model's log-likelihood function

$$\begin{aligned}\ell(\boldsymbol{\beta}, \lambda^2, \boldsymbol{g}, \sigma^2) = &-\frac{T}{2}\log(2\pi) - \frac{1}{2}\log|\lambda^2 \boldsymbol{C}_k + \sigma^2 \boldsymbol{I}_T| \\ &- \frac{1}{2}(\boldsymbol{y} - \boldsymbol{X\beta})'(\lambda^2 \boldsymbol{C}_k + \sigma^2 \boldsymbol{I}_T)^{-1}(\boldsymbol{y} - \boldsymbol{X\beta}),\end{aligned} \tag{2.23}$$

which is the basis for the LM tests for nonlinearity Under the null hypothesis $H_0 : \vartheta = \tilde{\vartheta}$ with $\tilde{\vartheta} = (\vartheta_1^{0\prime}, \tilde{\vartheta}_2')'$, the Lagrange multiplier statistic is given by $LM = \boldsymbol{s}(\tilde{\vartheta})'\mathcal{I}^{-1}\boldsymbol{s}(\tilde{\vartheta})$ where $\boldsymbol{s}(\tilde{\vartheta})$ is score function, and $\mathcal{I}$ means the information matrix. Hamilton's λ-test, denoted $\lambda_{\mathrm{H}}^{\mathrm{E}}$, is base on Hessian log-likelihood function information matrix $\mathcal{I}_{\mathrm{H}}$. Dahl and González-Rivera's λ-test, denoted $\lambda_{\mathrm{OP}}^{\mathrm{A}}$, is based on outer-product of the score function $\mathcal{I}_{\mathrm{OP}}$ information matrix. The LM statistic is $\chi^2 \sim (q)$ where q equals the number of restrictions under the null.

2.3.1 λ-test

λ test based on known covariance functions

There will be a unidentified nuisance problem when proceeding λ test under the null $H_0 : \lambda^2 = 0$. Hamilton deals with this by fixing $\boldsymbol{g}$. In other word, he assumes the complete knowledge of covariance matrix associated with the random field, when calculating the LM statistic. That is to say, we must fully know the covariance to the random filed by assumption. Due to the L_2 norm, the model (2.22) shows that $\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{X\beta}, \lambda^2 \boldsymbol{H}_k + \sigma^2 \boldsymbol{I}_T)$ and yields the log-likelihood function (2.23) for $\boldsymbol{C}_k = \boldsymbol{H}_k$. Furthermore, the score function is

$$\frac{\partial\, \ell(\boldsymbol{\beta}, \lambda^2, \boldsymbol{g}, \sigma^2)}{\partial \lambda^2}\bigg|_{\lambda^2=0,\boldsymbol{g};\boldsymbol{C}_k=\boldsymbol{H}_k} = \frac{1}{2\sigma^4}\left[\boldsymbol{u}'\boldsymbol{H}_T\boldsymbol{u} - \sigma^2 \mathrm{tr}(\boldsymbol{H}_T)\right],$$

and Hessian type information matrix $\mathcal{I}_{\mathrm{H}}$ can be expressed as

$$\mathcal{I}_{\mathrm{H}} = \begin{bmatrix} (2\sigma^4)^{-1}\mathrm{tr}(\boldsymbol{H}_T^2) & (2\sigma^4)^{-1}\mathrm{tr}(\boldsymbol{H}_T) & \boldsymbol{0}' \\ (2\sigma^4)^{-1}\mathrm{tr}(\boldsymbol{H}_T) & (2\sigma^4)^{-1}T & \boldsymbol{0}' \\ \boldsymbol{0} & \boldsymbol{0} & \sigma^{-2}\boldsymbol{X}'\boldsymbol{X} \end{bmatrix}.$$

So that, the $\lambda_{\mathrm{H}}^{\mathrm{E}}$ statistic, which is based on Euclidean distance and Hessian type information matrix, can be calculate as

$$\lambda_{\mathrm{H}}^{\mathrm{E}}(\boldsymbol{g}) = \frac{\hat{\boldsymbol{u}}'\boldsymbol{H}_T\hat{\boldsymbol{u}} - \tilde{\sigma}_T^2 \mathrm{tr}(\boldsymbol{M}_T\boldsymbol{H}_T\boldsymbol{M}_T)}{\left(2\mathrm{tr}\left\{[\boldsymbol{M}_T\boldsymbol{H}_T\boldsymbol{M}_T - (T-k-1)^{-1}\boldsymbol{M}_T\mathrm{tr}(\boldsymbol{M}_T\boldsymbol{H}_T\boldsymbol{M}_T)]^2\right\}\right)^{1/2}} \sim \chi^2(1), \tag{2.24}$$

where $\boldsymbol{M} = \boldsymbol{I}_T - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$.

Dahl and González-Rivera propose an alternative λ-test, $\lambda_{\mathrm{OP}}^{\mathrm{E}}$, based on $\mathcal{I}_{\mathrm{OP}}$, and derive it as TR^2 version statistic conditional on fully known covariance function. Fixing $\boldsymbol{g}$ likewise and calculate score function under the null, they are

$$s(\lambda^2)\Big|_{\lambda^2=0,\boldsymbol{g}} = \frac{\partial \ell(\boldsymbol{\beta}, \lambda^2, \boldsymbol{g}, \sigma^2)}{\partial \lambda^2}\bigg|_{\lambda^2=0,\boldsymbol{g}} = -\frac{1}{2\sigma^2}\tilde{\boldsymbol{x}}_1'\boldsymbol{\kappa},$$

$$s(\sigma^2)\Big|_{\lambda^2=0,\boldsymbol{g}} = \frac{\partial \ell(\boldsymbol{\beta}, \lambda^2, \boldsymbol{g}, \sigma^2)}{\partial \sigma^2}\bigg|_{\lambda^2=0,\boldsymbol{g}} = -\frac{1}{2\sigma^2}\tilde{\boldsymbol{x}}_2'\boldsymbol{\kappa},$$

where $\tilde{\boldsymbol{x}}_1 = \mathrm{vec}(\boldsymbol{C}_k)$, $\tilde{\boldsymbol{x}}_2 = \mathrm{vec}(\boldsymbol{I}_T)$, and $\boldsymbol{\kappa} = \mathrm{vec}(\boldsymbol{I}_T - \boldsymbol{u}\boldsymbol{u}'/\sigma^2)$. Since $\boldsymbol{\kappa}'\boldsymbol{\kappa}/T^2 \xrightarrow{\mathrm{p}} 1$, the TR^2 version of the LM statistic is

$$\lambda_{\mathrm{OP}}^{\mathrm{E}}(\boldsymbol{g}) = \frac{T^2}{2}\frac{\boldsymbol{\kappa}'\tilde{\boldsymbol{x}}(\tilde{\boldsymbol{x}}'\tilde{\boldsymbol{x}})\tilde{\boldsymbol{x}}\boldsymbol{\kappa}}{\boldsymbol{\kappa}'\boldsymbol{\kappa}} \sim \chi^2(1), \tag{2.25}$$

where $\tilde{\boldsymbol{x}} = (\tilde{\boldsymbol{x}}_1 : \tilde{\boldsymbol{x}}_2)$. We can obtain the statistic by the following procedure: (1) Estimate the model under the null and compute $\hat{\boldsymbol{u}} = \boldsymbol{y} - (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$ and $\hat{\sigma}^2 = \boldsymbol{u}'\boldsymbol{u}/T$. (2) Obtain the least squares estimate of $\boldsymbol{v}$, denoted $\hat{\boldsymbol{v}}$, from the auxiliary regression $\hat{\boldsymbol{\kappa}} = \phi_1\tilde{\boldsymbol{x}}_1 + \phi_2\tilde{\boldsymbol{x}}_2 + \boldsymbol{v}$, using $\hat{\boldsymbol{\kappa}} = \mathrm{vec}(\boldsymbol{I}_T - \hat{\boldsymbol{u}}\hat{\boldsymbol{u}}'/\hat{\sigma}^2)$. (3) Obtain the uncentered R^2 as $R^2 = 1 - \hat{\boldsymbol{v}}'\hat{\boldsymbol{v}}/\hat{\boldsymbol{\kappa}}'\hat{\boldsymbol{\kappa}}$. (4) Finally, the LM statistic exhibits as $\lambda_{\mathrm{OP}}^{\mathrm{E}} = T^2R^2/2$.

λ-test based on unknown covariance functions

If the parameterized covariance function is unknown, then Dahl and González-Rivera suggest that approximate the covariance function by Taylor expansion. In addition, we need to reassume the the random field to the covariance $m(\cdot)$ is continuous and differentiable with homogeneities and Gaussianness, and consider the permissible covariance function $\boldsymbol{C}_k^*$ in (2.10) where $\boldsymbol{C}_k^*(h_{ts}^*) = (1-h_{ts}^*)^{2k} 1_{(h_{ts}^* \leqslant 1)}$ for $h_{ts}^* = (1/2) d_{L_1}(\boldsymbol{g} \odot \boldsymbol{x}_t, \boldsymbol{g} \odot \boldsymbol{x}_s) = (1/2)\boldsymbol{r}_{ts}'\boldsymbol{g}$, and $\boldsymbol{r}_{ts} = \{|x_{t1} - x_{s1}|, |x_{t2} - x_{s2}|, \ldots, |x_{tk} - x_{sk}|\}'$. Then take the Taylor expansion on $\tilde{\boldsymbol{x}}_1 = \mathrm{vec}(\boldsymbol{C}_k)$ from auxiliary regression, and rewrite it as

$$
\begin{aligned}
\hat{\boldsymbol{\kappa}}_{ts} &= \phi_1 \tilde{\boldsymbol{x}}_{ts,1} + \phi_2 \tilde{\boldsymbol{x}}_{ts,2} + \boldsymbol{v}_{ts} \\
&= \phi_1 \left[\sum_{j=0}^{2k} \binom{2k}{j} h_{ts}^{*j} (-1)^j \right] 1_{(h_{ts}^* \leqslant 1)} + \phi_2 \tilde{\boldsymbol{x}}_{ts,2} + \boldsymbol{v}_{ts}.
\end{aligned}
$$

The aim is to avoid the nuisance problem on $\boldsymbol{g}$, when proceeding the λ-test. So that, under the assumption of L_1 norm, we can measure the nuisance parameter $\boldsymbol{g}$ as a linear function. And we replace the indicator function $1_{(h_{ts}^* \leqslant 1)}$ with a smooth function, like logistic function $1_{(h_{ts}^* \leqslant 1)} \approx (1 + \exp(-\gamma(1 - h_{ts}^*))^{-1}$ for fixed $\gamma \gg 0$. Furthermore, we can restate the auxiliary regression as

$$
\begin{aligned}
\hat{\kappa}_{ts} = \bar{\phi}_0 &+ \bar{\phi}_1 \sum_{i=1}^{k} g_i r_{ts,i} + \bar{\phi}_2 \sum_{i=1}^{k} \sum_{j \geqslant i}^{k} g_i g_j r_{ts,i} r_{ts,j} \\
&+ \bar{\phi}_3 \sum_{i=1}^{k} \sum_{j \geqslant i}^{k} \sum_{l \geqslant j}^{k} g_i g_j g_l r_{ts,i} r_{ts,j} r_{ts,l} + \ldots \\
&+ \bar{\phi}_{2k+2} \sum_{i=1}^{k} \sum_{j \geqslant i}^{k} \cdots \sum_{m}^{k} g_i g_j \ldots g_m r_{ts,i} r_{ts,j} \ldots r_{ts,m} + \phi_2 \tilde{x}_{2,ts} + v_{ts}, \qquad (2.26)
\end{aligned}
$$

where $\bar{\phi}_j$ is proportional to ϕ_1, that is $\bar{\phi}_j = c_j \phi_1$ with c_j being the proportionality parameter. The subindex ts attached to the vectors $\hat{\boldsymbol{\kappa}}$, $\tilde{\boldsymbol{x}}_2$, and $\boldsymbol{v}$ means the tsth entry/row in the respective vector for $t, s = 1, 2, \ldots, T$, and g_i and $r_{ts,i}$ denote the ith entry in the vector $\boldsymbol{g}$ and $\boldsymbol{r}_{st}$, respectively. By the new auxiliary regression (2.26), $\phi_1 = 0$ implies that $\hat{\kappa}_{ts} = \bar{\phi}_0$.[5] To rephrase it, $\phi_1 = 0$ eliminates the $\boldsymbol{g}$ component from the auxiliary regression, and circumvents the identification of $\boldsymbol{g}$ under the null. Finally we

[5] Since $\bar{\phi}_j$ is directly proportional to ϕ_1.

construct analogously the new λ-test

$$\lambda_{\text{OP}}^{\text{A}} = T^2 R^2 \sim \chi^2(q^*)$$

where $q^* = 1 + \sum_{j=1}^{2k+2} \binom{k+j-1}{k-1}$.

2.3.2 g-test

Under the null hypothesis $H_0 : \boldsymbol{g} = \mathbf{0}_k$, we have mentioned that λ can not be identified and stochastic processes become nonergodic, which is the crucial to the law of large number.[6] We modify the model (2.1) in different way, that is $y_t = \alpha_0 + \boldsymbol{\alpha}'\boldsymbol{x}_t + \lambda\tilde{m}(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t$, where $\tilde{m}(\boldsymbol{x}) = m(\boldsymbol{x}) - m(\mathbf{0}_k)$. Notice that $\tilde{m}(\mathbf{0}_k) = 0$ and $\tilde{m}(\boldsymbol{x})$ obeys Gaussian distribution, the model under the null becomes $y_t = \alpha_0 + \boldsymbol{\alpha}'\boldsymbol{x}_t$ which restores the ergodicity of y_t. The key of restoring the ergodicity is $\tilde{m}(\boldsymbol{x})$, thus the covariance $\tilde{\boldsymbol{C}}_k$ to the modified random field $\tilde{m}(\boldsymbol{x})$ is worthy to be discussed. Let $\tilde{\boldsymbol{C}}_k = \mathrm{E}[\tilde{m}(\boldsymbol{x})\tilde{m}(\boldsymbol{z})]$ be the covariance function that uniquely determines the random field $\tilde{m}(\boldsymbol{x})$. Calculate the $\tilde{\boldsymbol{C}}_k$ as

$$\begin{aligned}
\tilde{\boldsymbol{C}}_k(\boldsymbol{x}, \boldsymbol{z}) &= \mathrm{E}[m(\boldsymbol{x})m(\mathbf{0}_k)][m(\boldsymbol{z})m(\mathbf{0}_k)] \\
&= \mathrm{E}[m(\boldsymbol{x})m(\boldsymbol{z})] + \mathrm{E}[m(\mathbf{0}_k)m(\mathbf{0}_k)] - \mathrm{E}[m(\boldsymbol{x})m(\mathbf{0}_k)] - \mathrm{E}[m(\boldsymbol{z})m(\mathbf{0}_k)] \\
&= \boldsymbol{C}_k(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{C}_k(\mathbf{0}_k, \mathbf{0}_k) - \boldsymbol{C}_k(\boldsymbol{x}, \mathbf{0}_k) - \boldsymbol{C}_k(\mathbf{0}_k, \boldsymbol{z}), \qquad (2.27)
\end{aligned}$$

which $\tilde{\boldsymbol{C}}_k$ is called structure function, and is permissible.

Analogous to the $\lambda_{\text{OP}}^{\text{A}}$, we also want to propose a new $\boldsymbol{g}$-test, based on L_1 norm and denoted as $\boldsymbol{g}_{\text{OP}}$, which is free of the nuisance problem of λ parameter under the null. As the same token, we keep λ fixed and calculate its score function under the null. They are

$$s(g_i)|_{\lambda^2, \boldsymbol{g}=0} = -\frac{\lambda^2}{2\sigma^2}\tilde{\boldsymbol{x}}_i'\boldsymbol{\kappa}, \qquad i = 1, 2, \ldots, k, \qquad (2.28)$$

$$s(\sigma^2)\Big|_{\lambda^2, \boldsymbol{g}=0} = -\frac{1}{2\sigma^2}\tilde{\boldsymbol{x}}_{k+1}'\boldsymbol{\kappa}, \qquad (2.29)$$

[6] Consider the model (2.1) under the null $y_t = \alpha_0 + \boldsymbol{\alpha}'\boldsymbol{x}_t + \lambda m(\mathbf{0}_k)$, where $m(\mathbf{0}_k) \sim \mathcal{N}(0, 1)$. It becomes apparent that the model will be linear on $\boldsymbol{x}_t$, but y_t will be nonergodic since

$$\mathrm{E}(y_t y_{t-s}) = \begin{cases} \lambda^2 + \sigma^2 & \text{for } s = 0, \\ \lambda^2 & \text{for } s > 0. \end{cases}$$

where $\tilde{\boldsymbol{x}}_i = \partial\,\mathrm{vec}(\tilde{C}_k)/\partial g_i|_{g=0}$, $\tilde{\boldsymbol{x}}_{k+1} = \mathrm{vec}(\boldsymbol{I}_T)$, and $\boldsymbol{\kappa} = \mathrm{vec}(\boldsymbol{I}_T - \boldsymbol{u}\boldsymbol{u}'/\sigma^2)$. By the score function (2.28) and (2.29), we can derive the TR^2 version LM statistic of g-test with the auxiliary regression

$$\hat{\kappa}_{ts} = \sum_{i=1}^{k} \tilde{\phi}_1 \tilde{r}_{ts,i} + \tilde{\phi}_{k+1} \tilde{x}_{k+1,ts} + \tilde{\nu}_{ts}, \tag{2.30}$$

where $\tilde{r}_{ts,i} = -k(|x_{ti} - x_{si}| - |x_{ti}| - |x_{si}|)$, for $t, s = 1, 2, \ldots, T$. So that, the LM statistic is given as $g_{\mathrm{OP}} = T^2 R^2 \sim \chi^2(k)$.

Chapter 3

Empirical Study

3.1 Data Description and Uncertainty Measurement

We use monthly data of consumer price index (CPI) of the Four Dragon of East Asia, that is, Taiwan, Hong Kong, Singapore, and South Korea, to investigate the relationships between inflation and inflation uncertainty. The sample period are 1980:01～2002:12, 1985:01～2003:07, 1977:01～2003:07, and 1965:01～2003:08 for Taiwan, Hong Kong, Singapore, and South Korea, respectively. The data source comes from Taiwan Education AREMOS Data Bank.

Preliminarily we need to solve a problem that how to measure the inflation uncertainty? The traditional approach to investigate this issue is by the GARCH-type models. The merit of this model is that the inflation uncertainty is automatically constructed by the conditional heteroscedasticity estimate of the GARCH model. Because the flexible regression model cannot generate the conditional variance as the GARCH model, we need to construct a specific measure for the inflation uncertainty. Following Arize et al. (2000), we instead take the measurement of moving average standard deviation as our proxy for the inflation uncertainty.[1] The inflation uncertainty measurement is defined

[1]A fundamental problem of this measurement is that it is a "generated regressor variable" which might understate the true inflation uncertainty. However, Lo and Piger (2003) presents the estimated results between generated and ungenerated variables and find little difference between them. Hamilton's approach is a trad-off because GARCH model cannot catch the nonlinear relationships between inflation rate and inflation uncertainty, though, it avoids the generated regressor problem inherently.

as follows.

$$J_{t+m} = \left[\frac{1}{m} \sum_{i=1}^{m} (R_{t+i-1} - R_{t+i-2})^2 \right]^{1/2}, \tag{3.1}$$

where R is the nature logarithm of CPI, and m is the order of the moving average. In this study, we employ the order $m = 7$. Figure 3.1 graphs the inflation and inflation uncertainty of these economies.

3.2 Econometric Model

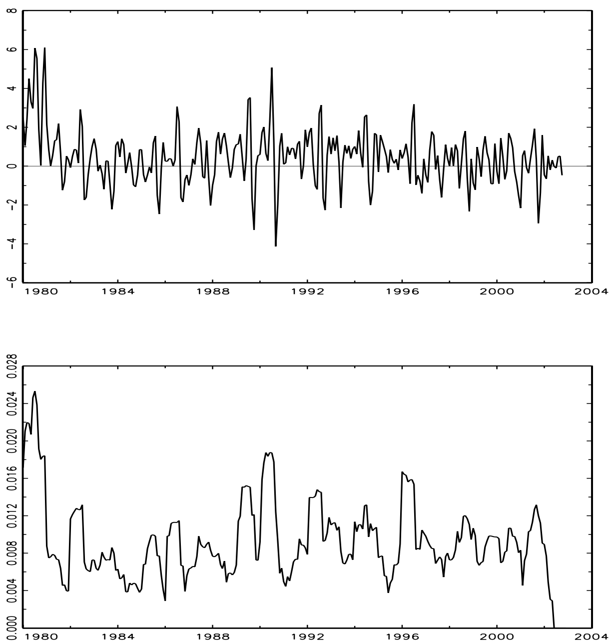
We employ Hamilton's flexible regression model to find the relationships between inflation rates and inflation uncertainty. We focus on two hypotheses. The first is "Friedman's hypothesis", that is, does higher rates of inflation increase higher inflation uncertainty? Another hypothesis is "Cukierman-Meltzer's hypothesis" which examines does higher inflation uncertainty cause higher level of inflation rates? As discuss in previous chapter on model specification, the empirical models for the two hypotheses are as follows

$$\sigma_{\pi_t} = \beta_0 + \sum_{j=1}^{q} \beta_j \sigma_{\pi_{t-j}} + \varphi \pi_t + \lambda_\sigma m(\boldsymbol{k} \odot \boldsymbol{z}_t) + \nu_t, \tag{3.2}$$

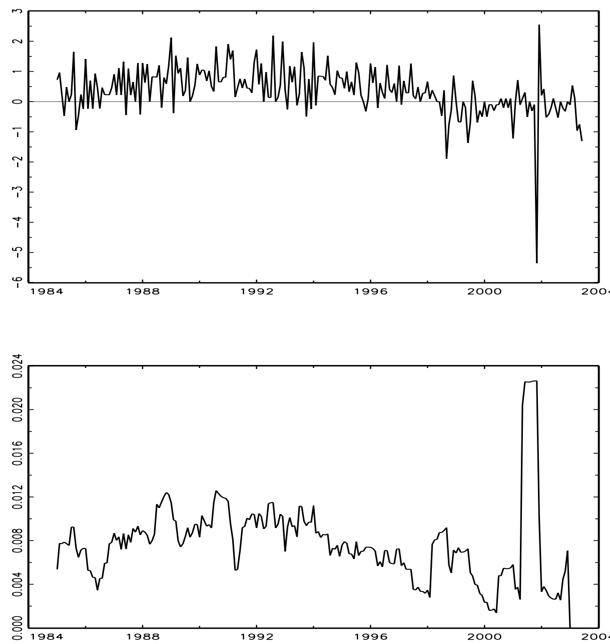
$$\pi_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \pi_{t-i} + \phi \sigma_{\pi_t} + \lambda_\pi m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t, \tag{3.3}$$

where $\boldsymbol{z}_t = \{\sigma_{\pi_{t-1}}, \sigma_{\pi_{t-2}}, \ldots, \sigma_{\pi_{t-q}}, \pi_t\}$, $\boldsymbol{x}_t = \{\pi_{t-1}, \pi_{t-2}, \ldots, \pi_{t-p}, \sigma_{\pi_t}\}$. Terms π_t and σ_{π_t} denote the inflation and inflation uncertainty, respectively. Terms q and p denote the optimal lag length of equation (3.2) and (3.3), respectively. If the estimate of φ in equation (3.2) is significantly different from zero, then it provides evidence that the inflation rates have linear effect on the inflation uncertainty. By the same token, if the estimate of ϕ is significantly different from zero, then the inflation uncertainty will exert effect on inflation rates. Instead of linear relation between inflation rates and inflation uncertainty, we catch the nonlinear relationships between them by Hamilton's flexible regression model. We show the detail empirical results in the following paragraphs.

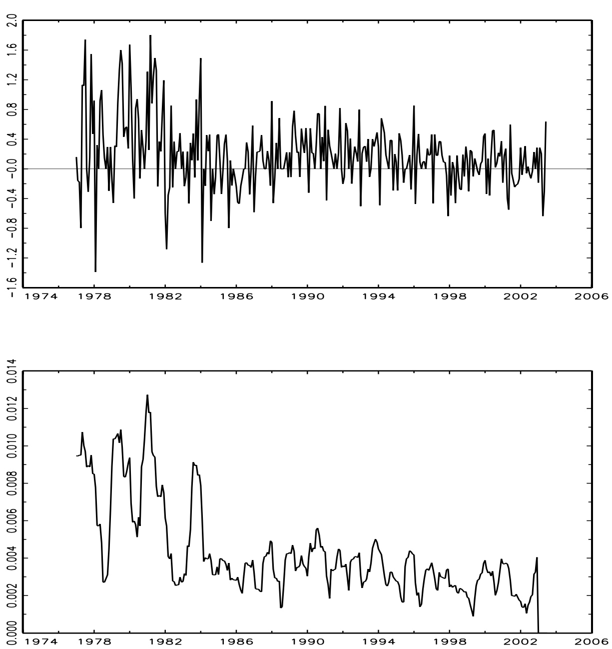
Before estimation, we select the optimal lag lengths of regressors in equations (3.2) and (3.3) by the Schwarz Bayesian criterion (SBC) instead of Akaike information cri-



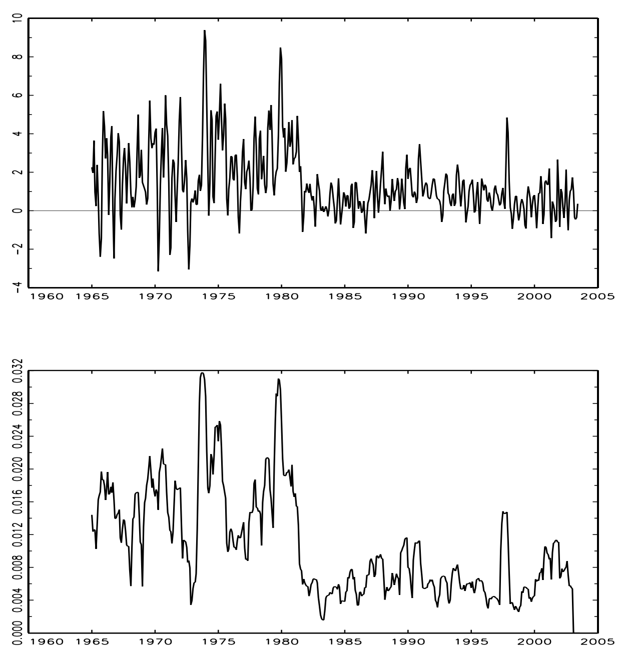
(a) Taiwan's inflation and inflation uncertainty



(b) Hong Kong's inflaiton and inflation uncertainty



(c) Singapore's inflation and inflation uncertainty



(d) Korea's inflation and inflation uncertainty

Figure 3.1: Figures of inflation and inflation uncertainty in four economies

Table 3.1: The results of the optimal lag lengths of regressors selected by SBC

	Taiwan	Hong Kong	Singapore	South Korea
$\sigma_{\pi t-1}$	−12.588	−12.547	−14.540	−12.654
$\sigma_{\pi t-2}$	−12.592*	−12.556*	−14.606*	−12.744*
$\sigma_{\pi t-3}$	−12.590	−12.539	−14.597	−12.730
$\sigma_{\pi t-4}$	−12.569	−12.514	−14.583	−12.720
π_{t-1}	0.031*	−0.216	−1.208	0.418
π_{t-2}	0.039	−0.211	−1.306	0.247
π_{t-3}	0.059	−0.391*	−1.389*	0.103*
π_{t-4}	0.057	−0.380	−1.371	0.105

Symbol * denotes the best selection by SBC

terion. The reason is based on Dahl and Gonzále-Rivera (2003), they mentioned that "moderate number of lags is recommended to guard against dynamic misspecification." Table 3.1 reports the results from AR(1) to AR(4). According to parsimonious principle, the final chosen model is picked up by choosing the minimum value of SBC. For equation (3.2), the optimal lags are two for all economies. As for equation (3.3), the optimal lags for Hong Kong, Singapore, and South Korea are three, while for Taiwan it is unity.

3.3 Empirical Analysis

3.3.1 Taiwan

Panel A in Table 3.2 presents the empirical results of equation (3.2) i.e., Friedman's hypothesis, and the nonlinear tests statistics. Several observations can be extracted from it. First, if the linear hypothesis $\lambda = 0$ is not rejected, then the regression (3.2) turns out to be linear since the nonlinear part in equation (3.2) disappears. From the table, it is clear that the $\lambda_{\mathrm{H}}^{\mathrm{E}}$, $\lambda_{\mathrm{OP}}^{\mathrm{E}}$, and $\lambda_{\mathrm{OP}}^{\mathrm{A}}$ statistics significantly reject the linear null hypothesis in favor of the nonlinear alternative. As a result, we may conclude that the relation is nonlinear. Second, the linear estimate of π_t is not significant at 5% level, it seems that Friedman's hypothesis does not hold for linear relation. Third, as for the nonlinear component, we can observe that the estimates of $\sigma_{\pi_{t-1}}^{\mathrm{TW}}$ and $\sigma_{\pi_{t-2}}^{\mathrm{TW}}$ are

Table 3.2: The estimated results of the linkage between inflation and inflation uncertainty in the case of Taiwan

(A) Friedman's hypothesis: $\sigma_{\pi_t} = \beta_0 + \sum_{j=1}^{q} \beta_j \sigma_{\pi_{t-j}} + \varphi \pi_t + \sigma[\zeta m(\boldsymbol{k} \odot \boldsymbol{z}_t) + \nu_t]$

β_0	β_1	β_2	φ	σ	ζ	k_1	k_2	k_3	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
0.004***	0.825***	−0.030	3.8e−4	0.001***	1.025***	−43.926	11.872	0.539***	0.001***	0.001***	0.001***	0.001***
(0.001)	(0.078)	(0.063)	(2.2e−4)	(7.9e−5)	(0.303)	(26.650)	(22.593)	(0.073)				

(B) Cukierman-Meltzer's hypothesis: $\pi_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \pi_{t-i} + \phi \sigma_{\pi_t} + \sigma[\zeta m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t]$

α_0	α_1	ϕ	σ	ζ	g_1	g_2	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
−0.693	0.019	78.946**	0.860***	−0.921	1.157***	215.451***	0.004***	0.028**	0.002***	0.024**
(0.592)	(0.140)	(39.416)	(0.108)	(0.716)	(0.067)	(16.251)				

Rejection of null hypothesis at 1%, 5%, and 10% level is indicated by ***, **, and *, respectively. The number in parenthesis is the standard error.

Table 3.3: The estimated results of the linkage between inflation and inflation uncertainty in the case of Hong Kong

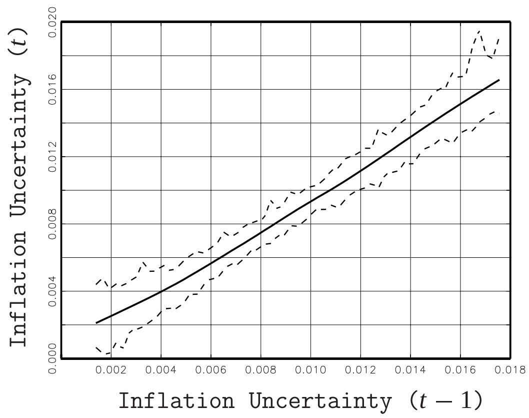
(A) Friedman's hypothesis: $\sigma_{\pi_t} = \beta_0 + \sum_{j=1}^{q} \beta_j \sigma_{\pi_{t-j}} + \varphi \pi_t + \sigma[\zeta m(\boldsymbol{k} \odot \boldsymbol{z}_t) + \nu_t]$

β_0	β_1	β_2	φ	σ	ζ	k_1	k_2	k_3	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
0.001	1.027***	−0.189***	−2.5e−5	0.002***	1.027	97.323	55.030	2.383	0.735	0.672	0.027**	0.132
(4.6e−4)	(0.068)	(0.069)	(1.9e−4)	(9.0e−5)	(0.203)	(245.590)	(456.070)	(2.062)				

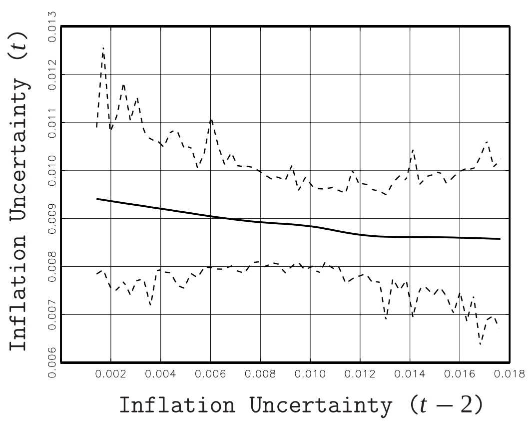
(B) Cukierman-Meltzer's hypothesis: $\pi_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \pi_{t-i} + \phi \sigma_{\pi_t} + \sigma[\zeta m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t]$

α_0	α_1	α_2	α_3	ϕ	σ	ζ	g_1	g_2	g_3	g_4	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
−0.266	0.658***	−0.530***	0.433***	38.323	0.539***	1.511***	0.261**	0.591***	0.839***	77.892***	0.005***	0.102	0.001***	0.002***
(0.456)	(0.111)	(0.127)	(0.127)	(32.283)	(0.059)	(0.416)	(0.113)	(0.175)	(0.185)	(28.032)				

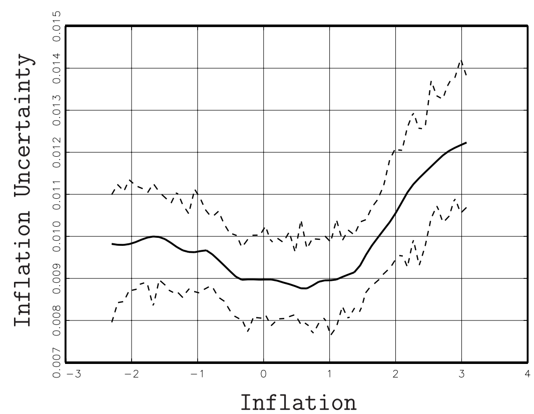
Rejection of null hypothesis at 1%, 5%, and 10% level is indicated by ***, **, and *, respectively. The number in parenthesis is the standard error.



(a) Effect of $\sigma_{t-1}^{\mathrm{TW}}$ on σ_t^{TW}



(b) Effect of $\sigma_{t-2}^{\mathrm{TW}}$ on σ_t^{TW}

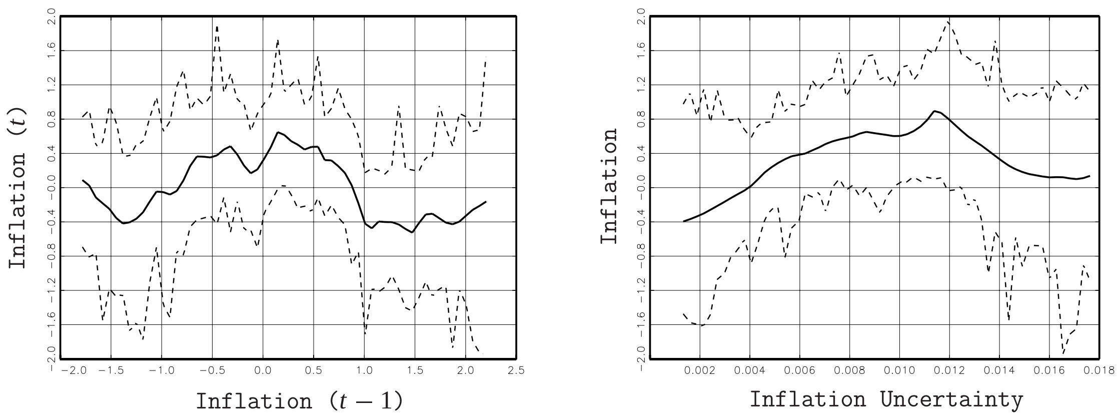


(c) Effect of π_t^{TW} on σ_t^{TW}

Figure 3.2: The effect of inflation on inflation Uncertainty—Taiwan.

insignificantly different from zero, in other words, $\sigma_{\pi_{t-1}}^{\mathrm{TW}}$ and $\sigma_{\pi_{t-2}}^{\mathrm{TW}}$ play no roles in the nonlinearity. By contrast, the nonlinear estimate of π_t^{TW} is statistically and significantly different from zero, suggesting that the nonlinearity seems to be mainly contributed by the π_t^{TW} variable. The result is evidenced by the linear test g_{OP} is significantly rejected.

As addressed in Hamilton (2001), given values of $\vartheta = \{\beta_0, \beta_1, \beta_2, \varphi, \zeta, k_1, k_2, k_3, \sigma\}$, we can calculate a value for equation (3.2) for any $\mathbf{z}^*$ of interest, which represents the econometrician's inference as the value of the conditional mean $\mu(\mathbf{z}^*)$ when the explanatory variables take on the value represented by $\mathbf{z}^*$ and when the parameters are known to take on these specified values. For example, Figure 3.2(a) plots the condi-



(a) Effect of π_{t-1}^{TW} on π_t^{TW} (b) Effect of σ_t^{TW} on π_t^{TW}

Figure 3.3: The effect of inflation uncertainty on inflation rate—Taiwan.

tional expectation function with respective to $\sigma_{t-1}^{\mathrm{TW}}$ holding $\sigma_{t-2}^{\mathrm{TW}}$, and π_t^{TW} constant, i.e., the figure plots $\hat{\mathrm{E}}[\mu(\sigma_{t-1}^{\mathrm{TW}}, \bar{\sigma}_{t-2}^{\mathrm{TW}}, \bar{\pi}_t^{\mathrm{TW}}) \mid \boldsymbol{Y}_T]$ as a function of $\sigma_{t-1}^{\mathrm{TW}}$ for $\bar{\sigma}_{t-2}^{\mathrm{TW}}, \bar{\pi}_t^{\mathrm{TW}}$ the sample mean for variable $\sigma_{t-2}^{\mathrm{TW}}$, π_t^{TW}, and $\boldsymbol{Y}_T$ the given sample observations on σ_t^{TW}, $\sigma_{t-1}^{\mathrm{TW}}$, $\sigma_{t-2}^{\mathrm{TW}}$, and π_t^{TW}. Solid line is the posterior mean with $N = 5,000$ Monte Carlo draws for specification. Dashed lines are the 95% confidence intervals. Figures 3.2(b) and (c) also plot the conditional expectation functions with respective to $\sigma_{t-2}^{\mathrm{TW}}$ and π_t^{TW}, respectively.

It is no doubt that the relationship between σ_t^{TW} and $\sigma_{t-1}^{\mathrm{TW}}$ is positive but linear, while the relationship between σ_t^{TW} and $\sigma_{t-2}^{\mathrm{TW}}$ is negative and linear but not significant, entirely consistent with the insignificant nonlinear estimates of k_1 and k_2 given in panel A of Table 3.2. Moreover, the functional form between σ_t^{TW} and π_t^{TW} (Figure 3.2(c)) displays the U shape, suggesting a nonlinear effect of the inflation on inflation uncertainty. A fact is that if deflation rates increase ($\pi_t^{\mathrm{TW}} < 0$), then the deflation uncertainty will increase. Likewise, if inflation rates increase ($\pi_t^{\mathrm{TW}} > 0$), then the inflation uncertainty will increase. Furthermore, inflation uncertainty is more sensitive to inflation in inflationary period than that in deflationary period, since the slope is asymmetric. Another interesting observation is that minimum level of inflation is at around 0.8%, suggesting that the best inflation target level to minimize the inflation uncertainty for the monetary authority is to set inflation rate at about 0.8%.

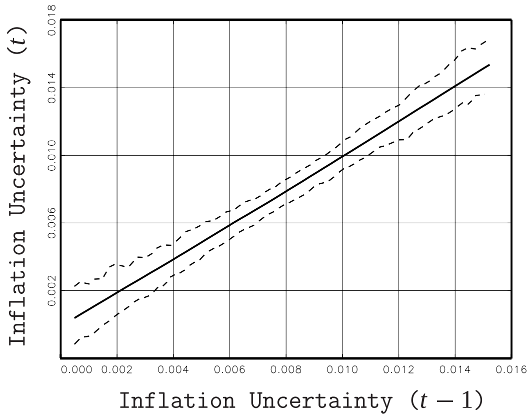
Overall, the linear estimate suggests that higher inflation rates have no effect on inflation uncertainty because φ is not significantly different from zero. However, from the estimate of nonlinear component k_3, the inflation rates exert significantly and positive effect on inflation uncertainty, suggesting that Friedman's hypothesis is supportable. This result provides evidence that we might make a bias conclusion if we ignore the important nonlinear component of the data.

Panel B of Table 3.2 summarizes the results of equation (3.3), which allow us to examine the reverse relationship that does higher inflation uncertainty cause higher rate of inflation (Cukierman-Meltzer's hypothesis)? First note that, again, the linear null hypothesis is significantly rejected by the λ test statistics at 5% level in favor of nonlinearity. Second, the linear estimate of ϕ is significantly and positively different from zero, suggesting that the inflation uncertainty has linear effect on inflation rates.

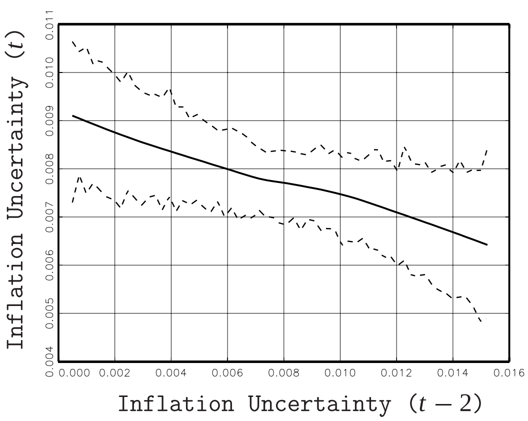
If we pay attention to the nonlinear component estimates, then we can find that the estimates of π_{t-1}^{TW} and σ_t^{TW} are significantly different from zeros. The results are consistent with the rejection of the g_{OP} test, and also evidenced by Figure 3.3(a) and 3.3(b), respectively. The effect of π_{t-1}^{TW} on π_t^{TW} displays an ambiguous pattern but does show the nonlinear relationships. The more interesting graphs is Figure 3.3(b), the effect of σ_t^{TW} on π_t^{TW} shows the inverted-U shape, suggesting a nonlinear effect of inflation uncertainty on inflation rates. It is interesting to note that there is a positive relation between inflation uncertainty and inflation rate (Cukierman-Meltzer's hypothesis holds) at a specific level of inflation uncertainty $\sigma_{\pi_t}^{\mathrm{TW}} = 0.012$. When the level of inflation uncertainty is higher that 0.012, then the pattern shows a negative relation in favor of Holland's hypothesis.

3.3.2 Hong Kong

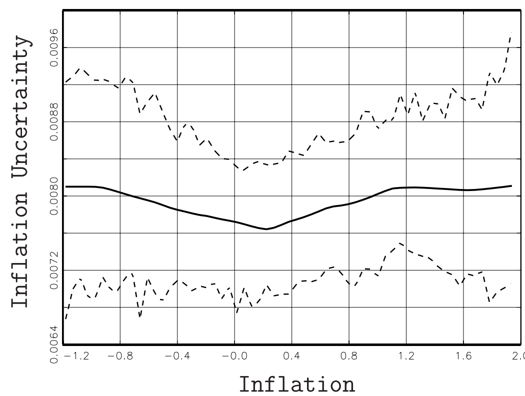
The empirical results for Hong Kong are summarized in panel A of Table 3.3. The linear null hypothesis cannot be rejected by the $\lambda_{\mathrm{H}}^{\mathrm{E}}$, $\lambda_{\mathrm{OP}}^{\mathrm{E}}$, and g_{OP} statistics, but can be rejected by $\lambda_{\mathrm{OP}}^{\mathrm{A}}$. The conflict consequences induce a difficulty in judging the nonlinear property of the data. For conservative reason, we double check the individual nonlinear component estimates, that is, k_1, k_2, and k_3. The estimates show that they are not significantly different from zero, which is consistent with the linear test result of g_{OP}. We conclude that there is no strong nonlinear evidence to support Friedman's hypothesis in Hong Kong.



(a) Effect of $\sigma_{t-1}^{\mathrm{HK}}$ on σ_{t}^{HK}



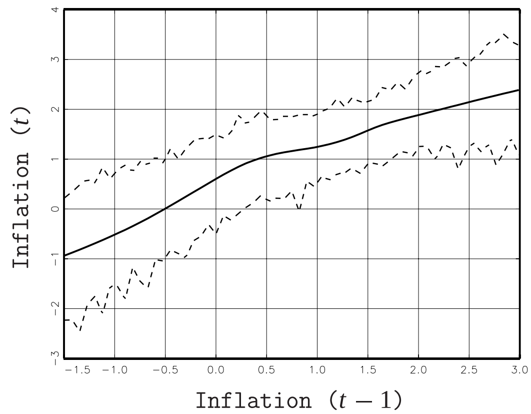
(b) Effect of $\sigma_{t-2}^{\mathrm{HK}}$ on σ_{t}^{HK}



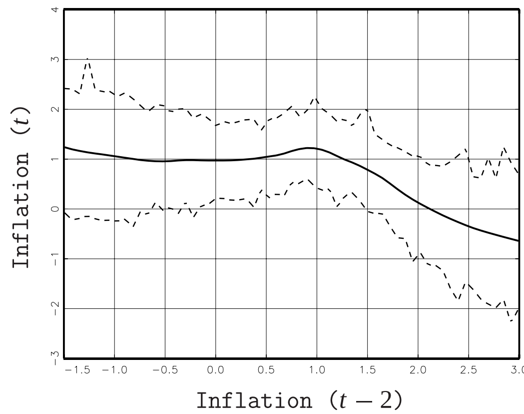
(c) Effect of π_{t}^{HK} on σ_{t}^{HK}

Figure 3.4: The effect of inflation on inflation uncertainty—Hong Kong.

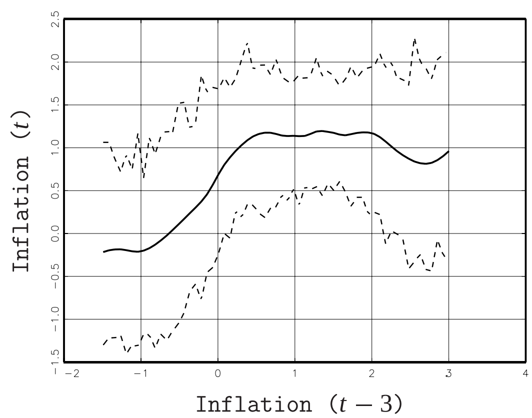
From panel A, it is no doubt that the linear estimate of $\sigma_{t-1}^{\mathrm{HK}}$ and $\sigma_{t-2}^{\mathrm{HK}}$ are positive and negative significantly different from zeros, respectively. While for the linear estimate of π_{t}^{HK}, it is insignificantly different from zero, suggesting that the inflation rate does not have linear effect on the inflation uncertainty in Hong Kong. The graphs of the conditional expectation functions with respective to $\sigma_{t-1}^{\mathrm{HK}}$, $\sigma_{t-2}^{\mathrm{HK}}$, and π_{t}^{HK} are put in Figure 3.4.The conditional expectation function of $\sigma_{t-1}^{\mathrm{HK}}$ on σ_{t}^{HK} is obviously positive linear sloped, while the condition expectation functions of $\sigma_{t-2}^{\mathrm{HK}}$ and π_{t}^{HK} on σ_{t}^{HK} are negative sloped and relative flat, respectively. Although they are not obvious straight, according to last paragraph expositions, we think them do not have nonlinear charac-



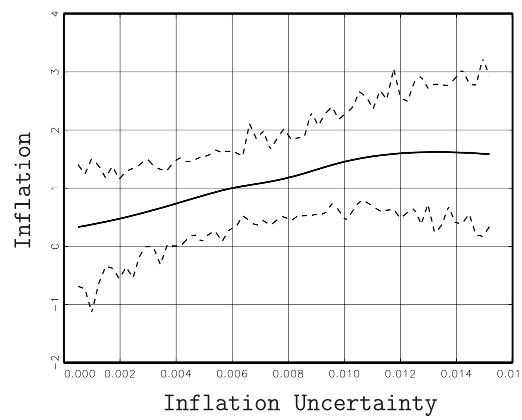
(a) Effect of π_{t-1}^{HK} on π_t^{HK}



(b) Effect of π_{t-2}^{HK} on π_t^{HK}



(c) Effect of π_{t-3}^{HK} on π_t^{HK}



(d) Effect of σ_t^{HK} on π_t^{HK}

Figure 3.5: The effect of inflation uncertainty on inflation rate—Hong Kong.

teristics.

Turning our attention to the Cukierman-Meltzer's hypothesis, empirical results are summarized in the panel B of Table 3.3. Two of the linear null hypothesis, λ-test (λ_{H}^{E} and λ_{OP}^{A}) and g-test, are significantly rejected at 1% level, suggesting there is nonlinear property in the data. However, the insignificant linear estimate of ϕ provides no evidence that higher inflation uncertainty will exert linear effect on higher inflation rates.

The linear test of g_{OP} is also rejected, which is consistent with the significant nonlinear component estimates of $\{g_1, \ldots, g_4\}$. Observe the conditional expectation function of π_t^{HK} with respectively to π_{t-1}^{HK}, π_{t-2}^{HK}, π_{t-3}^{HK}, and in Figure 3.5(a)–(c), they show the appearance of nonlinear relations. Figure 3.5(d) demonstrates that the Cukierman-Meltzer's hypothesis is held, that is, the higher inflation uncertainty raises higher rate of inflation.

Overall, the Friedman's hypothesis is not necessarily supportable in our model. By contrast, Cukierman-Meltzer's hypothesis is accepted for both linear and nonlinear components in Hong Kong.

3.3.3 Singapore

Panel A of Table 3.4 presents empirical results of equation (3.2) in the case of Singapore. First, the nonlinear test $\lambda_{\mathrm{H}}^{\mathrm{E}}$, $\lambda_{\mathrm{OP}}^{\mathrm{A}}$, and g_{OP} all reject the null hypothesis of the linearity of the model. We conclude that the relationship of equation (3.2) is nonlinear. Second, the linear estimate of π_t is not significant, implying that Friedman's hypothesis is not supportable. Third, as for the nonlinear component, estimates of $\hat{\boldsymbol{k}} = (413.886, 331.397, 2.245)'$ are statistically significant and different from zeros, indicating that all regressors including inflation π_t have nonlinear and positive effect on inflation uncertainty σ_t. We here find the nonlinear evidence that inflation does cause inflation uncertainty.

Analogously, Figure 3.6 displays the conditional expectation function of each variables. From Figure 3.6(a) and (b), the relationship between $\sigma_{t-1}^{\mathrm{SIG}}$ and σ_t^{SIG} is positive sloped; the relationship between $\sigma_{t-2}^{\mathrm{SIG}}$ and σ_t^{SIG} is negative sloped. Shed light on the relationship between inflation and inflation uncertainty, Figure 3.6(c) shows a slight U shape pattern where the higher inflation or deflation will exert higher inflation uncertainty.

Panel B of Table 3.4 summaries the results for equation (3.3). First, nonlinearity tests $\lambda_{\mathrm{H}}^{\mathrm{E}}$, $\lambda_{\mathrm{OP}}^{\mathrm{A}}$, and g_{OP} all reject the null hypothesis of linearity of the model at 1% significant level. It is intended to accept the nonlinear relation in equation (3.3). Second, the linear estimate of ϕ is significantly different from zero at 10% significant level, suggesting that weaker linear effect of inflation uncertainty affect rate of inflation.

By contrast, the nonlinear component estimate of σ_t^{SIG} is statistically and significantly different from zero, indicating that higher inflation uncertainty will have nonlinear effect on rate of inflation, i.e., Cukierman-Meltzer's hypothesis holds. Illustrate

Table 3.4: The estimated results of the linkage between inflation and inflation uncertainty in the case of Singapore

(A) Friedman's hypothesis: $\sigma_{\pi_t} = \beta_0 + \sum_{j=1}^{q} \beta_j \sigma_{\pi_{t-j}} + \varphi \pi_t + \sigma[\zeta m(\boldsymbol{k} \odot \boldsymbol{z}_t) + \nu_t]$

β_0	β_1	β_2	φ	σ	ζ	k_1	k_2	k_3	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
0.001**	1.126***	−0.261***	2.7e−4	5.2e−4***	1.287***	413.886***	331.397**	2.245***	0.036**	0.806	0.025**	0.001***
(4.4e−4)	(0.096)	(0.091)	(2.1e−4)	(3.9e−5)	(0.317)	(121.436)	(132.434)	(0.351)				

(B) Cukierman-Meltzer's hypothesis: $\pi_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \pi_{t-i} + \phi \sigma_{\pi_t} + \sigma[\zeta m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t]$

α_0	α_1	α_2	α_3	ϕ	σ	ζ	g_1	g_2	g_3	g_4	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
−0.534	0.777***	−0.493***	0.055	126.262*	0.337***	1.861***	0.210***	0.432**	0.091	399.728***	0.002***	0.093*	0.001***	0.001***
(0.533)	(0.113)	(0.163)	(0.077)	(72.703)	(0.034)	(0.668)	(0.092)	(0.162)	(0.064)	(38.033)				

Rejection of null hypothesis at 1%, 5%, and 10% level is indicated by ***, **, and *, respectively. The number in parenthesis is the standard error.

Table 3.5: The estimated results of the linkage between inflation and inflation uncertainty in the case of South Korea

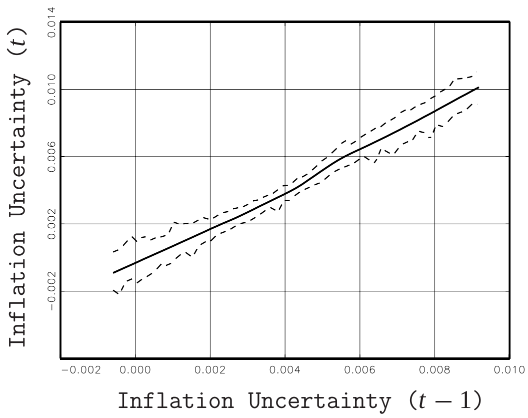
(A) Friedman's hypothesis: $\sigma_{\pi_t} = \beta_0 + \sum_{j=1}^{q} \beta_j \sigma_{\pi_{t-j}} + \varphi \pi_t + \sigma[\zeta m(\boldsymbol{k} \odot \boldsymbol{z}_t) + \nu_t]$

β_0	β_1	β_2	φ	σ	ζ	k_1	k_2	k_3	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
0.003***	1.255***	−0.434***	2.6e−4*	0.001***	0.981***	173.555***	175.302***	0.232***	0.041**	0.671	0.019**	0.130
(0.001)	(0.097)	(0.095)	(1.4e−4)	(8.2e−5)	(0.323)	(65.841)	(32.954)	(0.073)				

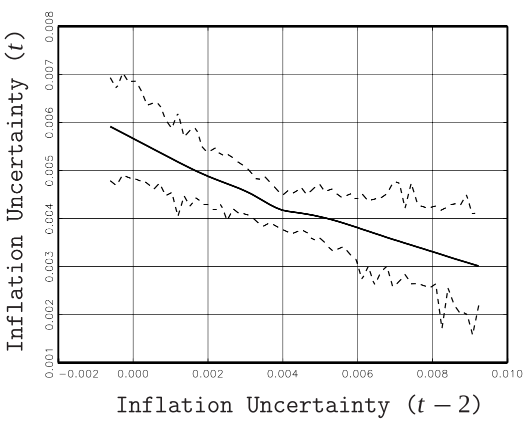
(B) Cukierman-Meltzer's hypothesis: $\pi_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \pi_{t-i} + \phi \sigma_{\pi_t} + \sigma[\zeta m(\boldsymbol{g} \odot \boldsymbol{x}_t) + \varepsilon_t]$

α_0	α_1	α_2	α_3	ϕ	σ	ζ	g_1	g_2	g_3	g_4	$\lambda_{\mathrm{H}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{E}}$	$\lambda_{\mathrm{OP}}^{\mathrm{A}}$	g_{OP}
−0.828***	0.842***	−0.647***	0.060	155.938***	0.243***	4.153***	0.747***	1.903***	−1.797***	120.757***	0.002***	0.017***	0.001***	0.002***
(0.211)	(0.062)	(0.073)	(0.060)	(16.994)	(0.098)	(1.855)	(0.105)	(0.267)	(0.345)	(26.764)				

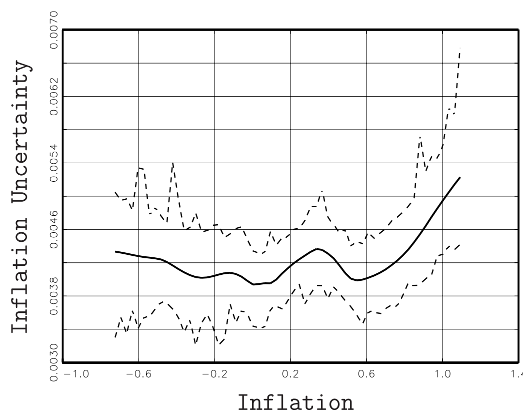
Rejection of null hypothesis at 1%, 5%, and 10% level is indicated by ***, **, and *, respectively. The number in parenthesis is the standard error.



(a) Effect of $\sigma^{\text{SIG}}_{t-1}$ on σ^{SIG}_{t}



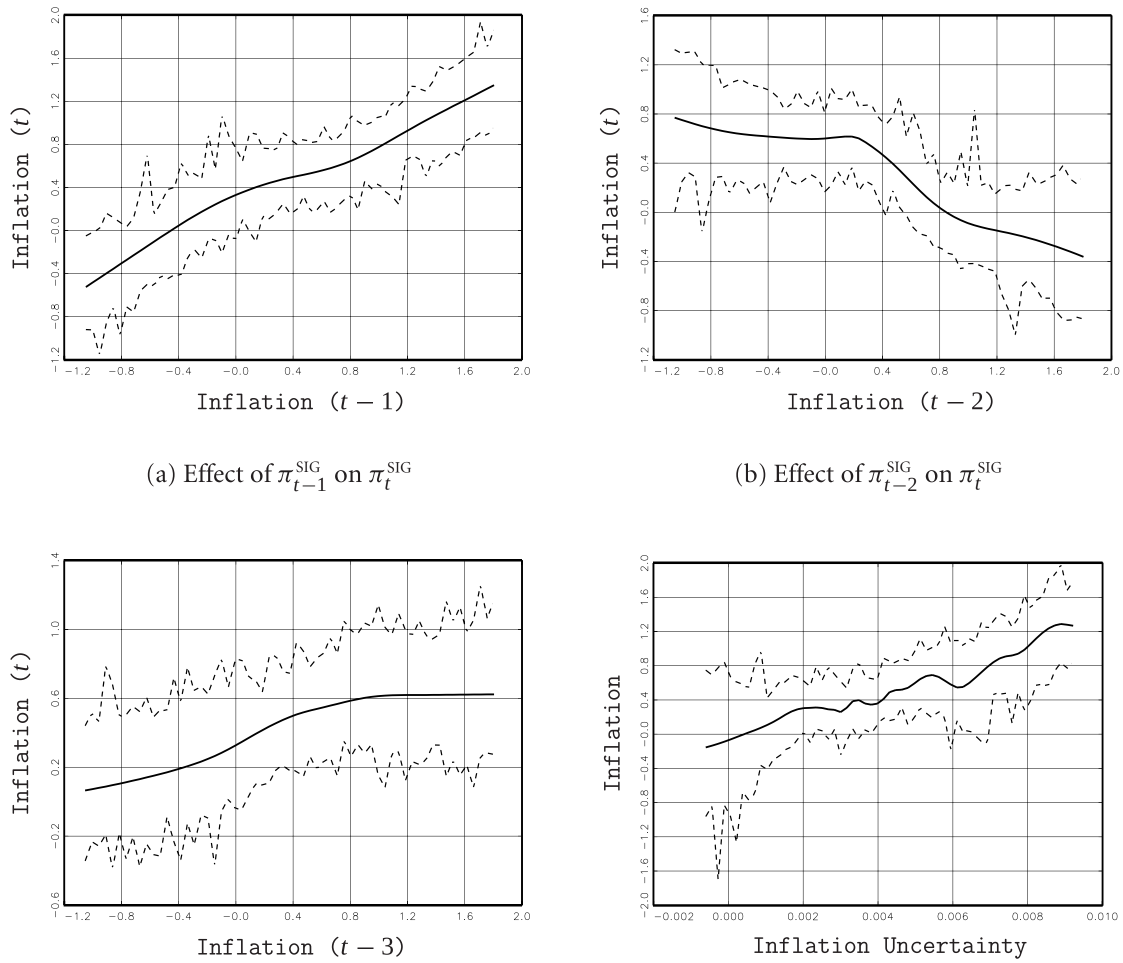
(b) Effect of $\sigma^{\text{SIG}}_{t-2}$ on σ^{SIG}_{t}



(c) Effect of π^{SIG}_{t} on σ^{SIG}_{t}

Figure 3.6: The effect of inflation on inflation uncertainty—Singapore.

again, Figure 3.7 displays the conditional expectation functions for each variables. Figure 3.7(a) and (c) demonstrate that the effect of π^{SIG}_{t-1} and π^{SIG}_{t-3} on π^{SIG}_{t} are both positive and nonlinear, respectively while Figure 3.7(b) exhibits that the effect of π^{SIG}_{t-2} on π^{SIG}_{t} is negative and nonlinear. Focusing on Figure 3.7(d), it shows that the relationship between inflation uncertainty and inflation is positive and nonlinear. In other words, it provides the evidence to support Cukierman-Meltzer's hypothesis that higher inflation uncertainty causes higher inflation uncertainty.



(a) Effect of π_{t-1}^{SIG} on π_t^{SIG}

(b) Effect of π_{t-2}^{SIG} on π_t^{SIG}

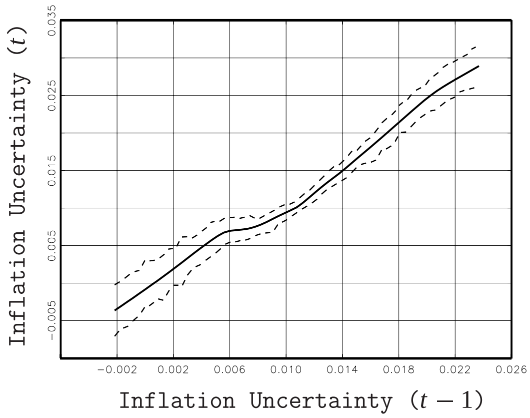
(c) Effect of π_{t-3}^{SIG} on π_t^{SIG}

(d) Effect of σ_t^{SIG} on π_t^{SIG}

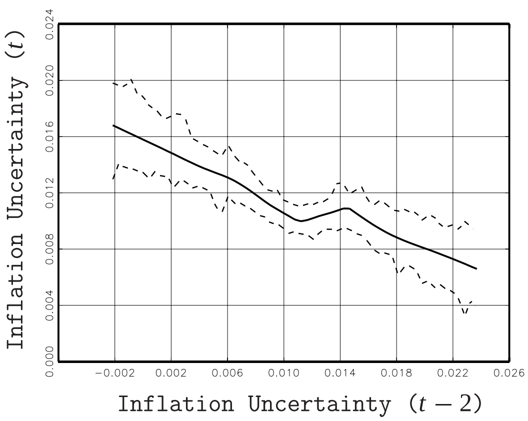
Figure 3.7: The effect of inflation uncertainty on inflation rate—Singapore.

3.3.4 South Korea

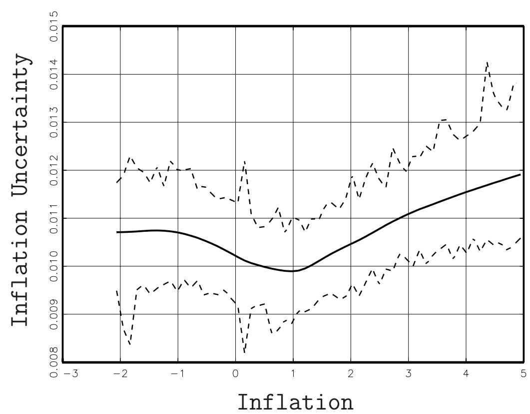
Panel A in Table 3.5 reports the empirical results of Friedman's hypothesis. A mixing result of linearity test is found. The $\lambda_{\text{OP}}^{\text{A}}$ and $\lambda_{\text{E}}^{\text{H}}$ reject linear hypothesis, while the $\lambda_{\text{OP}}^{\text{E}}$ and g_{OP} accept the linear null hypothesis. It becomes difficult for us to judge whether the variables contribute to nonlinearity. However, the nonlinear estimates $\hat{\boldsymbol{k}} = (173.555, 175.302, 0.232)'$ are all significantly different from zero, suggesting that there is the nonlinear properties in the model. Figure 3.8(a) and (b) plot that $\sigma_{\pi_{t-1}}^{\text{KR}}$ and $\sigma_{\pi_{t-2}}^{\text{KR}}$ have nonlinear positive and negative effect on $\sigma_{\pi_t}^{\text{KR}}$, respectively. As for Friedman's



(a) Effect of $\sigma_{t-1}^{\mathrm{KR}}$ on σ_t^{KR}



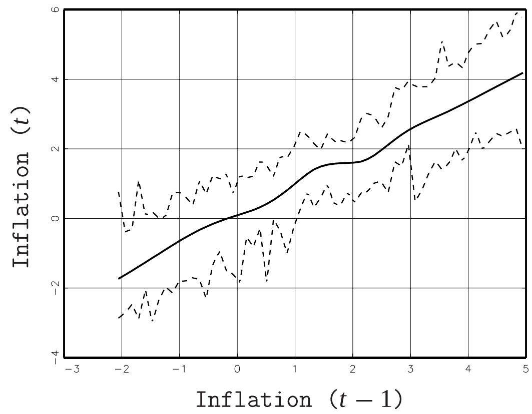
(b) Effect of $\sigma_{t-2}^{\mathrm{KR}}$ on σ_t^{KR}



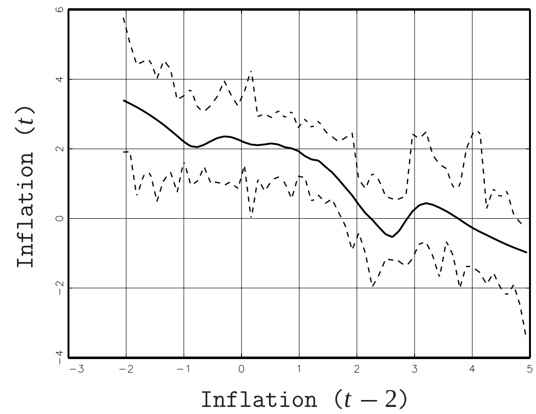
(c) Effect of π_t^{KR} on σ_t^{KR}

Figure 3.8: The effect of inflation on inflation uncertainty—South Korea

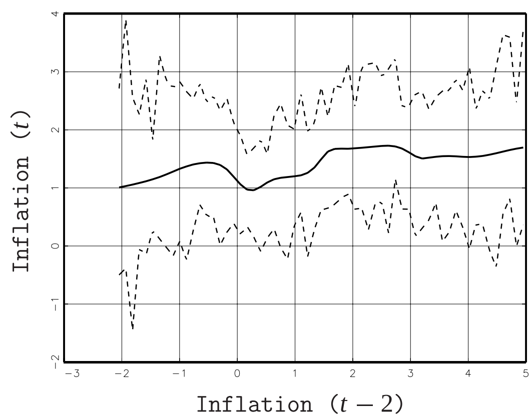
hypothesis, significance of nonlinear estimates of inflation provide the evidence in favor of it even though it is insignificant in linear estimate. Figure 3.8(c) plots the U shape relation between inflation and inflation uncertainty where the best target inflation rate to minimize inflation uncertainty is about 1%. Park (1995) examines Friedman's hypothesis using South Korea CPI and also find out the U-shaped relation between inflation and inflation uncertainty. He suggests that, under the assumption that inflation uncertainty has a negative effect on real economy, the policymaker should conduct the inflation policy in the range of the threshold level to prevent economic damage from inflation uncertainty



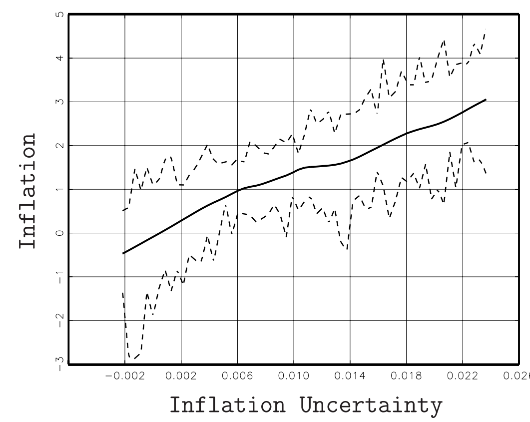
(a) Effect of π_{t-1}^{KR} on π_t^{KR}



(b) Effect of π_{t-2}^{KR} on π_t^{KR}



(c) Effect of π_{t-3}^{KR} on π_t^{KR}



(d) Effect of σ_t^{KR} on π_t^{KR}

Figure 3.9: The effect of inflation uncertainty on inflation rate—South Korea

In Panel B, it reports that the linear tests all reject the null hypothesis in favor of nonlinearity alternative. Furthermore, the linear and nonlinear estimates of inflation uncertainty both significantly and positively different from zero, indicating that increased inflation uncertainty raises inflation in favor of Cukierman-Meltzer's hypothesis. Figures 3.9(a) and (b) show positive and negative relationships between π_{t-1}^{KR} and π_t^{KR}, and π_{t-2}^{KR} and π_t^{KR}, respectively. Figure 3.9(c) shows that π_{t-3}^{KR} affects π_t^{KR} positively and nonlinearly. But the slope is more flatter relative to 3.9(a). Furthermore, Figure 3.9(d) shows the positive relation about Cukierman-Meltzer's hypothesis, consistent with our empirical results.

Table 3.6: The summary of empirical results of the relationship between inflation and inflation uncertainty

	Friedman's hypothesis			Cukierman-Meltzer's hypothesis		
	Linear	Nonlinear	Pattern	Linear	Nonlinear	Pattern
Taiwan	×	○	U	○	○	Inverted-U
Hong Kong	×	×	Flat	×	○	Positive sloped
Singapore	×	○	U	×	○	Positive sloped
South Korea	×	○	U	○	○	Positive sloped

3.4 Empirical Illustration and Policy Discussion

Table 3.6 illustrates a summary of our empirical study. In the linear estimates, Friedman's hypothesis is rejected for these four economies. After applying flexible nonlinear inference, we succeed to capture the nonlinear components to support Friedman's hypothesis except for Hong Kong. Furthermore, the relationships between inflation and inflation uncertainty all show an U shape. It can help the monetary authorities to target an specific level of rate of inflation to minimize inflation uncertainty to prevent economic damage. Another phenomenon from the U-shaped pattern is that the effect of inflation on inflation uncertainty is asymmetric.

On the other hand, by using the nonlinear inference, four economies provide overwhelming evidences in favor of Cuikerman-Meltzer's hypothesis. Three economies (Hong Kong, Singapore, and South Korea) show the positive effect of inflation uncertainty on inflation. Interestingly, Taiwan has a dramatic nonlinear pattern, inverted-U, in describing the relationship between inflation uncertainty and inflation. The effect of inflation uncertainty on inflation is, in general, positive. In details, *under* the specific (threshold) level of inflation uncertainty, the result supports Cuikerman-Meltzer's hypothesis; Instead, *over* the threshold level of inflation uncertainty, Cukierman-Meltzer's hypothesis is not accepted but in favor of Halland's hypothesis. The implications are that the monetary authorities of these three economies (Hong Kong, Singapore, and South Korea) prefer to behave a *opportunistic* policy to rise their economic growth (politically motivated expansionary policy). By contrast, the monetary authorities of Taiwan seem to prefer a discretionary policy. The Taiwan central bank will behave *stabilizing* policy to reduce economic harm when inflation uncertainty exceeds a threshold level.

Chapter 4

Conclusion

In this paper, we apply Hamilton (2001) flexible regression model to investigate the relationship between inflation and inflation uncertainty for four economies in the East Asia (Taiwan, Hong Kong, Singapore, and South Korea). Two hypothesis will be examined. One hypothesis is proposed by Friedman (1977), he argued that increased inflation could raise inflation uncertainty. The other hypothesis is provided by Cukierman and Meltzer (1986), they argued that high level of inflation uncertainty will cause higher rate of inflation. We find overwhelming statistical evidences that Friedman's hypothesis is hold except for Hong Kong. Interestingly, the nonlinearities look like U shape, implying that higher rates of inflation and deflation will raise inflation uncertainty. The pattern can help us to find a target rate of inflation to minimize inflation uncertainty and to reduce economic harm.

On the other hand, Cukierman-Meltzer's hypothesis is also evidenced for all four economies. Three economies (Hong Kong, Singapore, and South Korea) display positive relation in favor of Cukierman-Meltzer's hypothesis, while Taiwan has an inverted-U shape. Positive relation of Cukierman-Meltzer's hypothesis indicates that the monetary authorities prefer the *opportunistic* behavior to promote economic growth. On the contrary, in the case of Taiwan, under a specific level of inflation uncertainty, the Taiwan monetary authorities prefer *opportunistic* policy to rise economic growth. However, over a specific level of inflation uncertainty, Taiwan's monetary authorities alternatively behave an *stabilizing* policy to prevent economic damage from inflation uncertainty.

Appendix A

Distance Difinition

Li et al. (2003) mentioned and summarized Minkowski metric definition. The distance defined as Minkowski Distance (function) is such that

$$d_\rho(X, Y) = \left[\sum_{i=1}^{k} |x_i - y_i|^\rho \right]^{1/\rho} \tag{A.1}$$

where X and Y are presented by two ρ dimensional vectors $(x_1, x_2, \ldots, x_k)$ and $(y_1, y_2, \ldots, y_k)$, respectively. ρ in (A.1) is the Minkowski factor for the norm. $\rho = 2$ is so-called Euclidean Distance, it is

$$d_{L_2}(X, Y) = \sqrt{\sum_{i=1}^{k} (x_i - y_i)^2}. \tag{A.2}$$

When ρ is set as 1, it is Manhattan distance (or L_1 Norm). A variant Minikowski function, the weighted Minkowski distance function, shows that

$$d_\rho^w(X, Y) = \left[\sum_{i=1}^{k} w_i |x_i - y_i|^\rho \right]^{1/\rho} \tag{A.3}$$

where $w_i (i = 1, \ldots, k)$ is weighting coefficient.

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